DOI: 10.14714/CP106.2029

Map Projections *Don't* Have to be Hard

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INTRODUCTION

Discussing the years following the end of World War II, Raisz (1948) suggested in his *General Cartography* that this war had brought a new awareness of global problems and ushered in a new interest in cartography. He argued that this world-wide perspective also required greater attention be paid to the importance of map projections. While World War II concluded 80 years ago, global problems persist today, and an awareness of map projections is still relevant. However, despite Raisz's call for attention, public understanding of projections has not necessarily advanced since then.

Arguably, this lack of advancement is because, as Kessler and Battersby (2024) lament, map projections are hard—for many. Some of the reasons are that projections are inexorably tied to mathematics, their formulas contain mystifying parameters, distortion across the mapped area is misunderstood, and mapping software generally treats projections like black boxes (e.g., "project on the fly"). Coupled with these ideas is the acknowledgement that the projection literature is equally daunting to those not well versed in mathematics or the jargon associated with the field.

By extension, teaching projections is also hard. I have spent more than twenty-five years doing so, experimenting with approaches to improve student comprehension of projections in general and more specifically the projection process. Based on my experience, a fundamental point of student confusion rests in the difficulty of grasping how the projection process works, as well as how the choice of parameters controls the arrangement of the graticule and, ultimately, the distribution of distortion across the mapped area. To address this confusion, I developed an assignment that immersed students in the geometric construction of projections, instructing them in how to draw the graticule arrangements shown in Figure 1 for the (A) central cylindrical, (B) perspective conic, and (C) gnomonic projections. This assignment provided students with first-hand experience of using simple drawing tools (i.e., paper, pencil, ruler, protractor, and compass) to visualize the

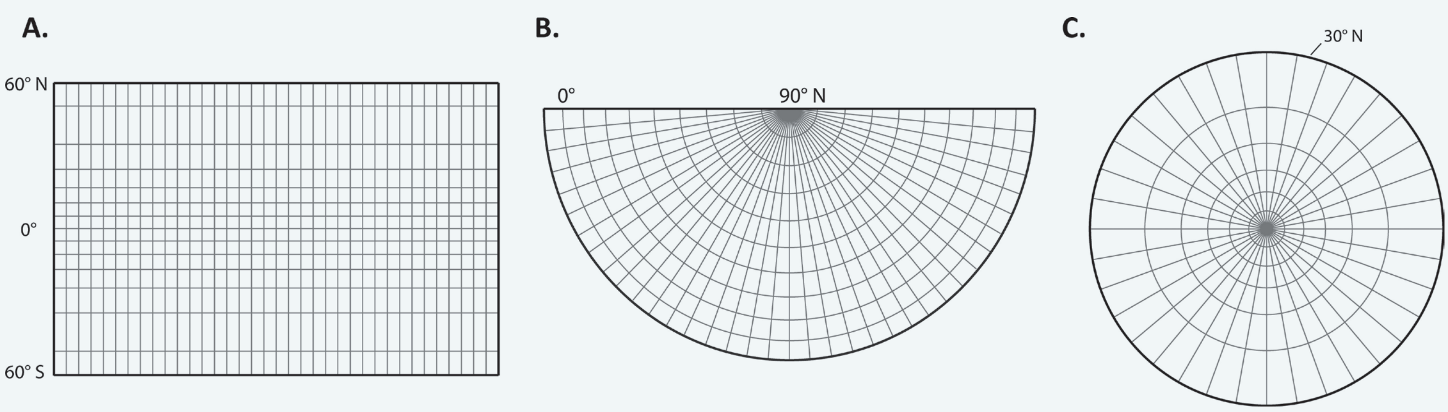

Figure 1. The graticule arrangement for the (A) central cylindrical, (B) perspective conic, and (C) gnomonic projections that will be geometrically constructed. The graticule spacing is 10°. The projections are not shown at the same scale.

projection process of transforming the round Earth onto a flat sheet of paper.

By following this assignment, students will accomplish three objectives that will help increase their comprehension of projections. First, students will explore the role that manual geometric construction methods play in plotting lines and curves representing meridians and parallels on projections. Second, students will visualize how simple geometrical relationships influence the graticule's appearance and final map scale. Third, students will connect projection parameters (e.g., the choice of a standard parallel) with how those parameters control the projection process. Once completed, students will recognize that projections *don't* have to be hard. In fact, this assignment will foster a new understanding of how elementary and fun projections can be.

ELEMENTS OF THE GEOMETRIC CONSTRUCTION PROCESS

Historically, projections were classified as either perspective or non-perspective (Deetz and Adams 1944). Snyder (1981) states that perspective projections "usually involve the geometric projection of points on the earth's surface onto a plane with all lines of projection passing through a common intermediate point . . . the perspective map simulates the earth as viewed from any point in space or from within the earth" (149). Non-perspective projections cannot be described as such. The process of manually plotting a perspective projection is often referred to as "geometric construction." Table 1 lists several well-known projections that are perspective.

The perspective projection process involves three elements: a reference globe, a projection plane, and a point of projection. The reference globe is a conceptually reduced version of Earth set to a size that will produce a map at the final desired map scale. In Figure 2, the reference globe is shown along the bottom of the figure with the poles at the top and bottom. The projection plane (the horizontal line) is tangent to the reference globe and will receive the tracings of the parallels and meridians. Depending on the desired graticule arrangement, this plane could also be secant to and passing through the reference globe. The point of projection can be thought of as the viewing point from which Earth's lines of latitude and longitude are projected. The point of projection can be positioned at various locations with respect to the reference globe (at a finite or infinite distance from the projection plane). In Figure 2, the point of projection is (A) at the center of, (B) on the opposite side of, and (C) at infinite distance from the reference globe. Based on the relationship between these three elements, the arrangement of the graticule can be controlled. For example, in Figure 2, the point of projection creates unique spacings of lines of latitude, producing the gnomonic, stereographic, and orthographic projections.

Azimuthal
General vertical perspective Gnomonic Stereographic Orthographic *Other Variations:* Lowry (1825) Fischer (1850) James (1857)
Conic
Perspective conic *Other Variations:* Braun (1867) Lidman (1876)
Cylindrical
Central cylindrical *Other Variations:* Gall "stereographic" (1885) Lambert cylindrical equal area (1772)

Table 1. Common perspective projections.

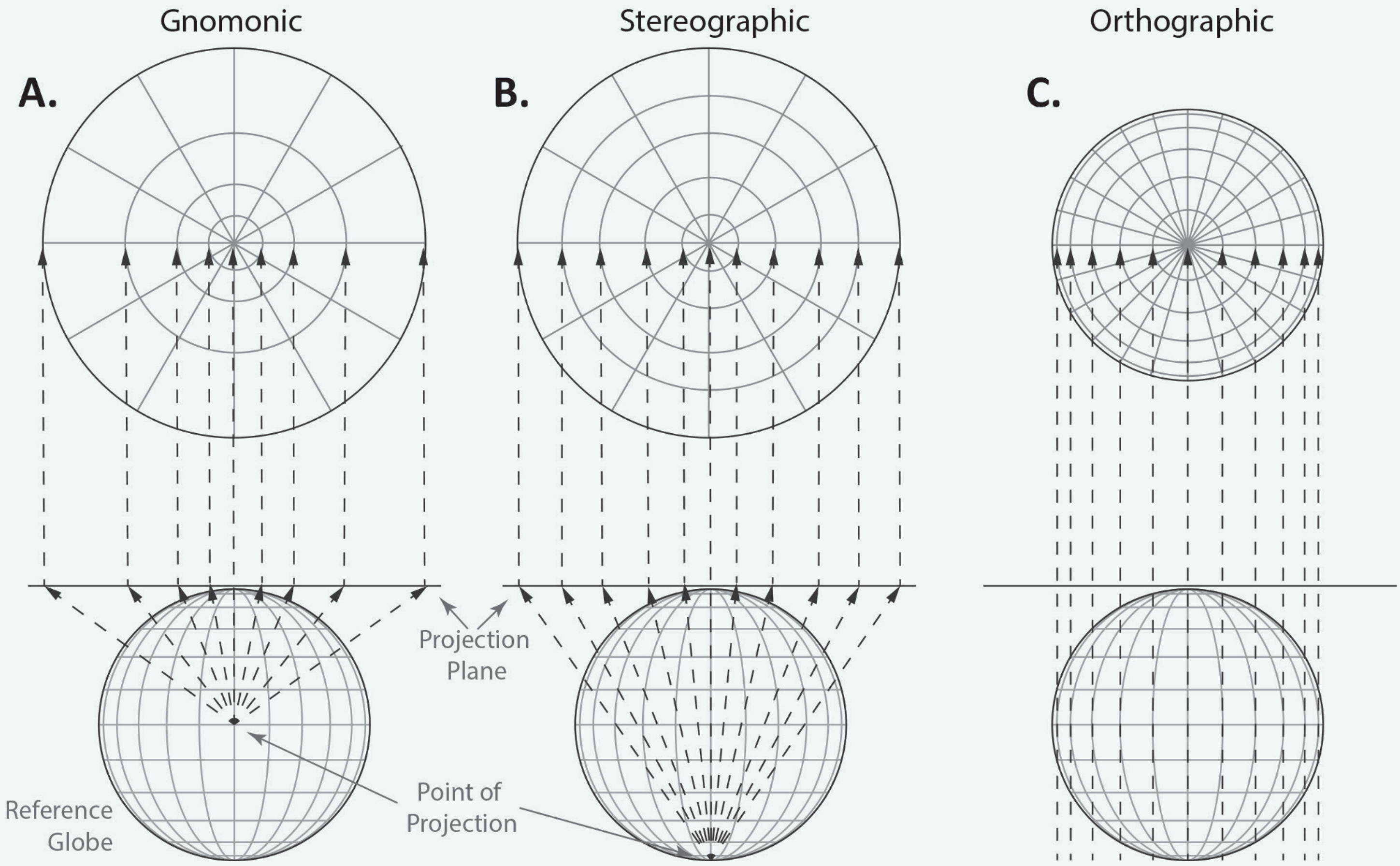

Figure 2. A comparison between different points of projection and the resulting arrangement of the graticule for the (A) gnomonic, (B) stereographic, and (C) orthographic projections. Adapted from Figure 8.3, in *Thematic Cartography and Geographic Visualization*, 4th edition. Slocum et al. 2023, CRC Press.

THE GEOMETRIC CONSTRUCTION PROCESS

This assignment geometrically constructs three projections (perspective cylindrical, perspective conic, and gnomonic) through the basic process described above, in which lines are drawn directly from a reference globe to a projection plane (a tangent 2D surface) forming the lines representing parallels and meridians (Figure 1). The perspective cylindrical will be centered along the equator, the perspective conic will be centered along 40° north latitude, and the gnomonic will be centered on a pole. To prepare for the construction process, the next section describes the scaling of the reference globe according to the final map scale.

SCALING THE REFERENCE GLOBE

The following three steps are needed to set the reference globe's size. First, we will start with the perspective cylindrical projection to determine the desired length of the line representing the equator. Second, based on the length of that line, we compute the representative fraction for the final map scale. Third, we calculate the size of the reference globe. Having determined the size of the reference globe for this projection, that same size will be used for the perspective conic and gnomonic projections so that their graticule arrangements can be visually compared according to the same map scale.

Step 1: Determine the Length of the Equator

The length of the equator can be set to any value that is commensurate with the dimensions of the sheet of paper that will receive the graticule. Here, we will assume an 11″×17″ sheet. From a mathematical standpoint, realize that drawing the equator at a specific length for the cylindrical projection sets the overall map scale. However, from a construction standpoint, the length of the line drawn to represent the equator will necessarily control the number of lines of latitude north and south of the equator that will appear (a longer equator line results in fewer lines of latitude fitting on the paper). I recommend setting the equator's length at 36 cm (≈14") which will allow parallels up to and including 60° north and south of the equator to be plotted.

Step 2: Determine the Final Map Scale

Equation 1 shows a simple mathematical relationship that can be used to solve for the final map scale, expressed as a representative fraction (*RF*). The *RF* is a ratio between a distance measured on the map (the *map distance)*, and the equivalent measurement on Earth's surface (the *earth distance*).

$$RF = \frac{\textit{map distance}}{\textit{earth distance}} \tag{1}$$

The 36 cm line representing the equator is the *map distance*. I previously established this 36 cm line to control the number of lines of latitude appearing above and below the equator. However, one can also set the length of line representing the equator to a predetermined overall map scale or RF. The corresponding *earth distance* is derived from Earth's mean equatorial circumference, which is 40,075,017 meters.[1] To simplify the calculation by using like units (cm), next we convert the circumference to 4,007,501,700 cm. Substituting these values into Equation 1 yields Equation 2, and an *RF* of 0.000000008983153277. Then, we take the reciprocal of the value, expressing it conventionally as 1:111,319,490 (Equation 3).

$$0.00000000898315277 = \frac{36\text{ cm}}{4{,}007{,}501{,}700\text{ cm}} \tag{2}$$

$$RF = 1{:}111{,}319{,}490 = (0.00000000898315277)^{-1} \tag{3}$$

Step 3: Calculate the Size of the Reference Globe

Having determined the *RF* (1: 1:111,319,490), we calculate the size of the reference globe. Earth has a mean radius of 6,378.137 km.[2] Substituting this value into Equation 1 as *earth distance* along with the *RF*, we get Equation 4. After converting the radius to its centimeter equivalent, we can rearrange to solve for the *map distance* (the radius of the reference globe) in Equation 5. The result in Equation 6 shows that a reference globe will have a radius of 5.73 cm. At this scale, all three projections will individually fit onto separate 11"×17" sheets of paper.

$$0.00000000898315277 = \frac{\textit{map distance}}{6{,}378.137\text{ km}} \tag{4}$$

$$\textit{map distance} = 0.00000000898315277 \times 637{,}813{,}700\text{ cm} \tag{5}$$

$$\textit{map distance} = 5.73\text{ cm} \tag{6}$$

THE CENTRAL CYLINDRICAL PROJECTION

The central cylindrical perspective projection has no known origin, but the transverse aspect was developed by Welch in the first half of the 1800s (Snyder 1993). Geometrically, the construction projects the globe onto a tangent or secant plane from a point of projection on the equatorial plane opposite a given meridian. In our case, we will geometrically construct the projection so that the point of projection is placed at the center of the reference globe and the projection plane will be tangent to the equator. Four steps are needed to complete this construction, illustrated in Figures 3A–3D. During the construction process, measure as accurately as possible and draw your lines with care.

1. Draw the reference globe so that its perimeter falls along one of the 8.5"×11" paper's short edges. Mark off 10° divisions along the reference globe's perimeter (Figure 3A). This interval creates a 10° parallel spacing in the final projection. Other parallel spacings can be created by marking the globe at a different interval. Position the reference globe so that its 0° mark (the tangent point) touches the left edge of the 11"×17" sheet of paper (Figure 3A). This tangent point marks the location of the equator.

2. Project individual lines from the center of the reference globe (the point of projection) passing through each 10° interval marked along the perimeter of the reference globe and intersecting the left-edge of the 11"×17" sheet of paper (Figure

1. Value from NASA's **Goddard Space Flight Center**.

2. Value from NASA's **Goddard Space Flight Center**.

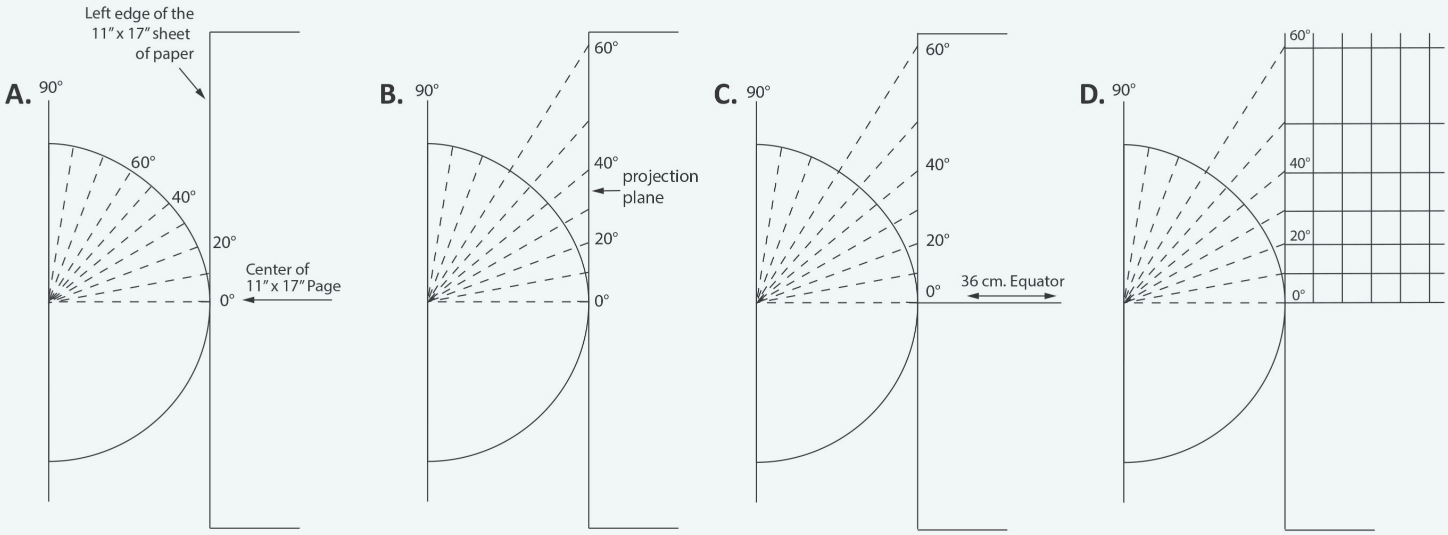

Figure 3. Four steps showing the geometric construction of the perspective cylindrical projection, using a 10° graticule spacing.

3B). Mark and labels these 10° intersections which indicate the position of the parallels.

3. Extend a horizontal line from 0° across the 11"×17" sheet of paper, representing the equator. This line should be 36 cm long from the tangent point (Figure 3C).

4. Draw the parallels and the meridians.

   a. At the marks previously located along the left edge of the 11"×17" sheet, draw horizontal lines representing the parallels north of the equator. Each line will be parallel to, and the same length as, the equator. Once completed, repeat the process for the parallels south of the equator.

   b. To draw meridians, recall that our equator is 36 cm long, and covers the 360° circumference of Earth. To match the 10° spacing of the lines of latitude for this projection, we will also plot meridians at a 10° spacing. Given the total range of meridians to be 360°, a 10° spacing would require that 36 meridians be plotted. Plotting 36 meridians along an equator that is 36 cm in length would require that each meridian be spaced 1 cm apart. Armed with this information, construct each of the 36 meridians as lines perpendicular to the parallels to represent these longitudinal distances. Figure 3D illustrates the initial drawing of the parallels and meridians north of the equator. See Figure 1A for the completed appearance of this cylindrical projection.

Note the spacing of the parallels on this projection. The spacing of the parallels increases from the equator poleward. For latitudes greater than 60° N/S, the line would be drawn beyond the top and bottom edges of the paper. In fact, the pole would be drawn at an infinite distance from the equator (i.e., the pole is projected along a path which parallels the tangent line).

A PERSPECTIVE CONIC PROJECTION

Next, we geometrically construct the perspective conic projection. This projection also has no recognized origin; although Snyder (1993) offers that Colles (1794) may have been the first to use the projection. While we placed the projection plane tangent to the reference globe at 0° in the central cylindrical projection, for the perspective conic, we position the reference globe to be tangent to a specific line of latitude. This line of latitude will become the standard line on the map, with no distortion. Arbitrarily, we use the fortieth parallel, but any other parallel would work, and would change the appearance of the graticule—mainly the spacing of the parallels. Five steps are needed to complete this task, which are illustrated by Figures 4A–4E.

1. Draw a reference globe with 10° intervals marked along its perimeter (Figure 4A). This division will create a graticule with a 10° parallel spacing. Note that the positioning of the reference globe with the 11"×17" sheet of paper is different than what appears in Figure 3A in two ways. First,

the reference globe is tangent to the 11"×17" sheet of paper at 40˚, making 40˚ latitude the tangent point. Second, the 40˚ tangent point should be set about 2.5 cm below the top-to-bottom center of the 11"×17" sheet of paper. Label the tangent point as 40˚ and then number the other ten-degree distances, from 50˚ to 90˚ above this point, and from 30˚ to 0˚ (the equator) below it.

2. Project, from the center of the reference globe and passing through each of these previously labeled points, individual lines to the projection plane (Figure 4B). Once projected, label each point along the projection plane.

3. Construct the circular arcs representing the parallels (Figure 4C). Center a compass on the tangent line the point representing the pole (90˚). Set the compass to the distance of the next parallel along the projection plane—80˚. Draw an arc representing this parallel, centered at 90˚ and extending through a half circle (180˚). Draw similar arcs for the other parallels at distances from the 90˚ point indicated along the projection plane. Draw as many circular arcs as will fit on the 11"×17" sheet of paper.

4. Mark off the distance of the meridians (every 10˚) along the 40˚ parallel (Figure 4D). Since this projection is developed along the fortieth parallel, the spacing of the meridians must be set at a distance appropriate for this latitude. To determine this distance, use **this webpage** to compute the length of one degree of longitude along 40˚ latitude. On the webpage, enter 40 in the Latitude textbox and press the Calculate button. One degree of longitude along the 40˚ parallel is 85,394 meters (8,539,400 cm). Every degree will be 0.076 cm apart (Equations 7–8 show the calculations). Thus, drawing a meridian every 10˚ would require spacing them 0.76 cm apart (0.076 × 10), representing a spacing of approximately 854 km on Earth.

$$0.00000000898315277 = \frac{\textit{map distance}}{8{,}539{,}400 \text{ cm}} \tag{7}$$

$$\textit{map distance} = 0.00000000898315277 \times 8{,}539{,}400 \text{ cm} = 0.076 \text{ cm} \tag{8}$$

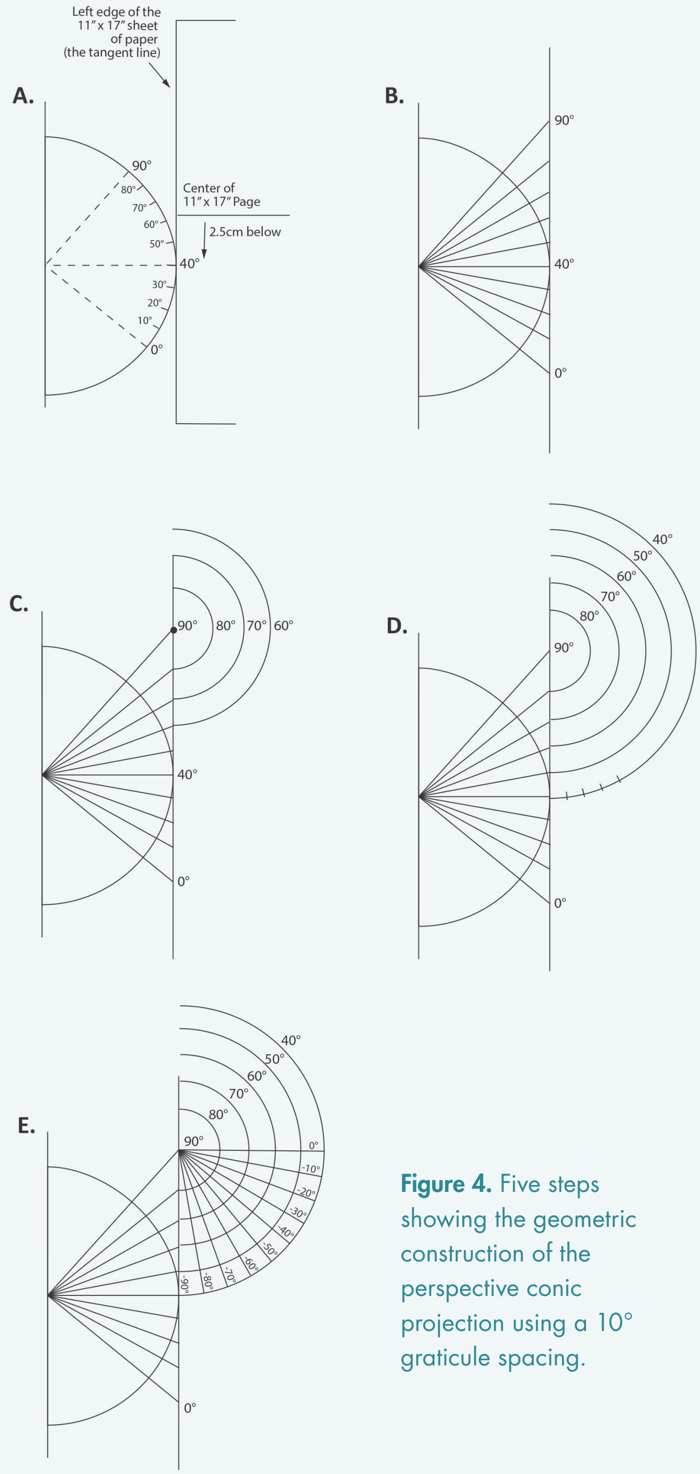

Figure 4. Five steps showing the geometric construction of the perspective conic projection using a 10° graticule spacing.

5. Mark these 0.76 cm distances along the 40˚ parallel. Then draw, straight lines from the pole through these marks constructing the meridians creating a 10˚ spacing (Figure 4E). See Figure 1B for the completed appearance of this conic projection.

THE GNOMONIC PROJECTION

Finally, the gnomonic projection will be constructed, which is one of the oldest geometric projections and was probably developed by Thales of Miletus, about 580 BCE (Snyder 1993). This projection is useful for navigational purposes, as it represents any great circle route anywhere on the projection as straight line. While the example provided here constructs the projection centered on a pole, Snyder (1949) provides a discussion of the geometric construction of the equatorial aspect of this graticule. The gnomonic projection represents less than a hemisphere. Constructing this perspective projection requires four steps (Figures 5A–5D).

1. Draw a reference globe at the center of the 11"×17" page. Next, draw a series of lines from its center creating 10° intervals that are marked along its perimeter (Figure 5A). Note that these intervals are numbered differently than what is shown in Figure 4A.

2. Draw a projection plane running top to bottom on the 11"×17" page, tangent to the reference globe at the 90° point. Extend lines from the reference globe's center to the projection plane (Figure 5B).

3. Mark the location of one of the poles on the projection plane, at 90° (Figure 5C). Draw, using a compass, a series of circles representing the parallels at distances from this point. Each circle is centered on 90° and passes through one of the distances marked along the projection plane.

4. Draw the meridians at a 10° spacing. Use a protractor to divide this circular network into thirty-six 10° segments along the outermost parallel. Starting at the pole, draw successive lines connecting the pole to the 10° marks along the outermost parallel, mapping the meridians (Figure 5D). See Figure 1C for the completed appearance of this gnomonic projection.

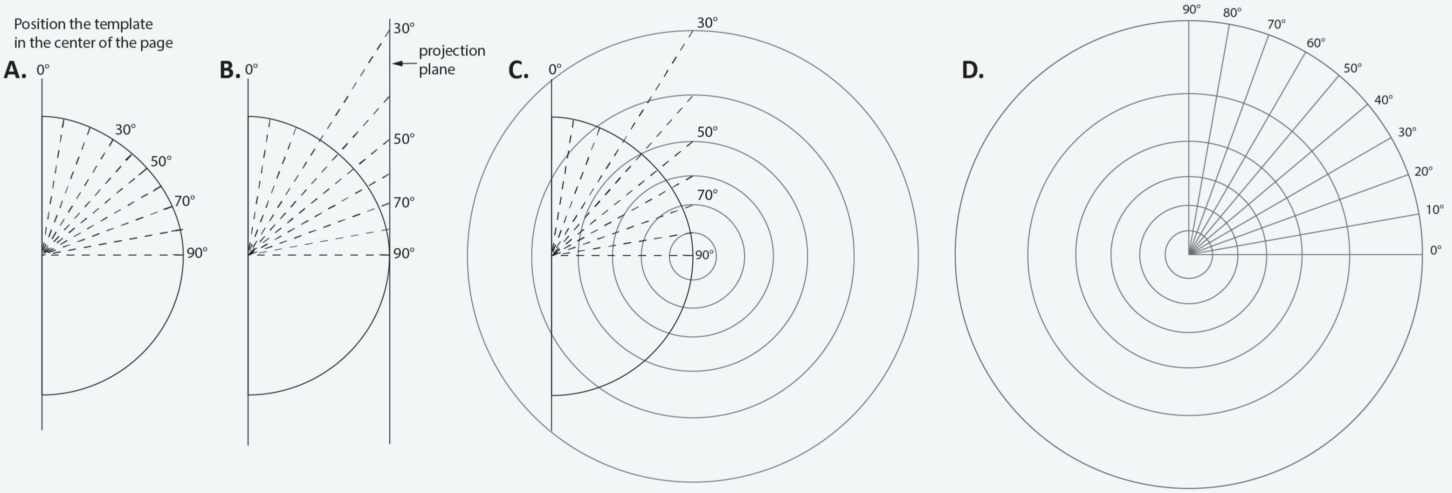

Figure 5. Four steps showing the geometric construction of the polar centered gnomonic projection using a 10° graticule spacing.

RESOURCES ON GRAPHICAL CONSTRUCTION OF MAP PROJECTIONS

Unfortunately, modern cartography texts do not discuss or provide instruction in geometric construction techniques. In a survey of projection content found in English-language cartography textbooks, I reported (2018) that references to geometric construction disappeared after 1970. Looking at textbooks before 1970, Raisz's *General Cartography*,[3] Robinson's *Elements of Cartography*,[4] and Birch's *Maps, Topographical and Statistical*[5] discussed

3. Inclusive of the 1st (1938) and 2nd (1948) editions.

4. Inclusive of the 1st (1953), 2nd (1960), 3rd (1969), and 4th (1978) editions.

5. Inclusive of the 1st (1949) and 2nd (1964) editions.

perspective projections, explained the geometric construction process, and illustrated that process. These authors focused on the familiar perspective cylindrical, conic, and azimuthal projections that are covered in this article.

Beyond textbooks, four map projection–specific books also highlighted geometric construction. Deetz and Adams's *Elements of Map Projection*[6] discussed and illustrated the construction process of the three projections presented in this assignment in addition to the stereographic and orthographic, Hinckley (1942) presented a summary of a variety of projections constructed geometrically. Of particular interest in this work is the construction of the gnomonic projection on three cubes. Hoffmeister (1946) provided a well-illustrated and descriptive commentary on the construction process for both the general classes of perspective projections and oblique aspects of the orthographic and stereographic projections. McDonnell (1979) included a discussion and examples of geometric construction of perspective projections. Additionally, numerous worked examples of constructing projections through simple trigonometry calculations are included. Investigating how simple trigonometric functions can be used to draw more accurate graticules for other projections would be a logical next step in understanding the projection process and how the parameters in these functions influence the appearance of the graticule. In a more recent text, Fenna (2007) includes a chapter titled "Shine a Light: Litteral projections" where the geometric construction process of perspective projections is illustrated and discussed in considerable detail.

REFLECTION

I, like many of my colleagues who teach cartography, have transitioned from manual techniques to computer-based methods. While this transition has been exciting, my experience shows that students equally enjoy and benefit from "hands-on" assignments. From my perspective, part of the "hands-on" appeal of geometrically constructing projections is that students visually experience and control how the projection process unfolds. This manual exercise is in contrast with having students click through a computer interface, while not knowing how the projection process unfolds. Presumably, if students derive enjoyment by working through this assignment, a positive learning experience may result. In fact, research offers some insights into how "fun" can be a part of the learning process. Lucardie (2014) reported that having fun and experiencing enjoyment throughout the learning experience were motivators to learn domain knowledge and skills. In addition, fun and enjoyment were key elements that encouraged students to concentrate, and thus better absorb the material. Erickson (2020) reported that students reported greater situational interest, motivation, and engagement when they participated in hands-on activities compared to those who were in a more traditional lecture-based learning environment. While I have no scientific evidence to support the idea that this particular geometric construction assignment improves learning about projections, I can state emphatically that students genuinely enjoyed working with the manual drafting tools to construct projections. They have expressed to me, in their end of the semester course evaluations, that they enjoyed working with this assignment as a break from the computer-based assignments, and would like to see more "hands on" work.

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