Beyond Graduated Circles: Varied Point Symbols for Representing Quantitative Data on Maps

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Graduated point symbols are viewed as an appropriate choice for many thematic maps of data associated with point locations. Areal quantitative data, reported by such enumeration units as countries, are frequently presented with choropleth maps but are also well suited to point symbol representations. Our objective is to provide an ordered set of examples of the many point-symbol forms used on maps by showing symbols with linear, areal, and volumetric scaling on repeated small maps of the same data set. Bivariate point symbols are also demonstrated with emphasis on the distinction between symbols appropriate for comparison (separate symbols) and those appropriate for proportional relationships (segmented symbols). In this paper, the variety of point symbol use is described, organized, and encourage, as is research on these varied symbols and their multivariate forms.

INTRODUCTION

Much cartographic research has been conducted on apparent-value scaling and perceptions of graduated point symbols. However, discussion of practical subjective elements of point symbol design and construction is limited, especially for the wide variety of point symbols for representing multivariate relationships that are now feasible with modern computing and output. The primary purpose of the paper is to suggest innovative graduated symbol designs and to examine useful combinations for comparing variables and representing proportional relationships. This paper was inspired by the bivariate symbol designs of hundreds of students that have taken introductory cartography courses with Cindy Brewer (at San Diego State and Penn State). These students have been creative and thoughtful in the many ways they have found to represent multivariate map data, with few choosing the old standby of graduated circles when given the flexible design capabilities of the illustration software.

"... graduated point symbols were the most common quantitative symbolization choice."

Although thematic atlases have long made use of a wide variety of point symbols, cartography textbooks emphasize use of graduated circles. Mersey (1996) surveyed the qualitative and quantitative point symbols used in eight recently published atlases and found that graduated point symbols were the most common quantitative symbolization choice (36 percent; with 59 percent of 36 being circles or sectored circles). In comparison, choropleth maps were used for 24 percent of the quantitative representations. Mersey notes that, despite the prevalence of point symbol mapping, choropleth maps are the mainstay of most microcomputer-based GIS programs. All of the 13 she reviewed offered choropleth mapping, with four of the 13 not able to produce graduated symbol representations. Muehrcke (1996), citing Abler (1987), refers to choropleth mapping as "a cartographic abomination that GIS will swiftly kill off" (p. 272) and then bemoans its continued use. Although GIS has enabled production of more sophisticated representations, it has also made it easier (for far more people) to produce choropleth maps as well.

Cartograms and dasymetric mapping offer alternatives to choropleth mapping, but here the more popular point symbol representations will be examined. Point symbols are appropriate for quantitative data at point locations and for areal units. They are not affected by the physical size of enumeration units (large units produce large symbols on choropleth maps) and can be used to illustrate a spatial distribution while maintaining recognizable geographic boundaries (unlike contiguous cartograms). Widely spaced point symbols in regions of large enumeration units (such as counties of the western U.S.) produce the appropriate sense of sparse data and they contrast well with dense clusters of symbols in areas of small units (such as eastern U.S. counties). Thus, point symbols preserve the spatial structure of the entire map without the problems associated with choropleth maps or cartograms.

The paper begins with a review of literature on graduated symbols to update the reader on this long research tradition in cartography and psychology. The remainder of the paper is devoted to a description of the variety of symbol designs possible and the nuances in symbol designs that make them appropriate for different types of bivariate map data. A wide variety of symbols are illustrated with the hope that these examples will inspire and encourage the authors of maps and GIS displays to produce a wider variety of engaging and enlightening data displays well-suited to communicating and exploring their geographic information.

LITERATURE REVIEW

Psychophysical Study of Graduated Circles

Apparent-value scaling and graduated circles have received a great deal of attention from cartographic researchers. The circle was, and still is, the most widely used graduated point symbol. Flannery's 1956 and 1971 work firmly established apparent-value scaling as a topic of research and discussion for the cartographic community.

Cox (1976) concluded that apparent value scaling "... turned out to be inadequate as a remedy for the underestimation of symbol size ratios" (p. 73). His research examined legend construction and anchor effects associated with circle and square size estimation. He found that using a legend containing a small, mid-size, and large symbol (relative to the mapped data) led to the most accurate estimates. His findings supported Dobson (1974), who stated "a review of psychophysical research on proportional circle symbols indicates that readers can interpolate between circle sizes but that extrapolation is quite difficult" (p. 53). Chang (1977) confirmed Cox's findings with the results of a similar study comparing size estimation using ratio and magnitude estimation methods.

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Macmillan et al. (1974), Teghtsoonian (1965), Maddock and Crassini (1980), and Shortridge and Welch (1980) examined the effects of the specific wording of the request that subjects estimate circle magnitudes. The latter three experiments were not entirely applicable to cartography because the standard for comparison needed to be remembered when estimations were made; whereas normally the "standard" would be simultaneously available in a map legend. Macmillan et al. tested the use of instructions to subjects with a standard present during estimations. They found that instructions for use of a fitting strategy improved the accuracy of subjects' size judgments.

By the mid-1980s, authors became increasingly skeptical about the usefulness of apparent-value scaling and shifted research away from correction by exponent. Griffin (1985) was particularly critical of its usefulness, stating:

> Perceptual rescaling may have merits for particular types of map user, or for the generation of immediate visual impressions, though both cases remain to be proven. Such rescaling places increased demands on

map space and, in the experimental context, promotes an increase in the variability of subject responses. (p. 35)

In analyzing intra- versus inter-subject variation in circle size estimation, Griffin pointed out the ineffectiveness of apparent-value scaling for increasing the estimation accuracy of subjects who performed poorly. He also noted that apparent-value scaling impaired those subjects whose estimates were most accurate. In contrast to Griffin's results, Olson's (1975) earlier work with circle-size and dot-density estimation examined the interaction between apparent-value scaling and training in the form of practice and feedback about correct answers (for commentary see Williams, 1977, and Olson, 1978). She concluded that the combination improves both the accuracy and dispersion of subject estimates, unlike results for either scaling or training alone. Assuming that a map legend functions as training in a limited way by providing example sizes with associated data values, apparent-value scaling may produce improvement in graduated-circle estimation for complete map-reading contexts that is not seen in simpler experimental contexts.

Interactive and animated mapping facilitates examination of multiple symbol scalings. Slocum and Yoder (1996) described student use of Visual Basic to animate series of graduated-circle maps. One approach was to permit interactive selection of scaling values from square-root areal scaling to linear scaling of diameters, which enhanced spatial pattern by greatly exaggerating relative circle areas.

Heino (1991) noted that graded rather than graduated symbols may be the best method of displaying quantitative data on a map. Heino argued that since graduated point symbols were meant as visual abstractions, graded symbols sufficiently convey the intended message without creating the problems associated with graduated symbol perception. Dent (1996) also discussed range-graded circles in his textbook. His discussion included Meihoefer's (1969) ten circle sizes that were "consistently discriminated by his subjects" (Dent, p. 174) as well as his own untested set of graded circles. Similarly, Monmonier (1977) recommended a regression-based method of circle scaling intended to improve visual correlations between maps by essentially imposing a shared minimum and maximum circle size on the data representations of two (or more) maps to be compared. His approach to scaling permitted accurate value estimations from legend examples, but correct size ratios between symbols were not maintained.

This shift in research away from apparent scaling was supported by authors such as Worth (1989) who discussed, as Flannery had, some of the problems with approaching cartographic research as a pure science. He noted that much cartographic research borrows from psychology because both fields are highly subjective, and stated that, "By their very nature, experimental tests in cartography must involve many subjective decisions, and although we must do our best to apply the scientific method, subjectivity will always be involved" (p. 152).

". . . much psychophysical study eliminates spatial structure from our perception of thematic figures . . ."

Petchenik (1983) offered an in-depth discussion of cartographic research and the problems associated with limiting ourselves to psychophysical study. She explained that much psychophysical study eliminates spatial structure from our perception of thematic figures, and quoted Chang (1980) who stated, "The stimulus-response relationship for circles is fairly complex, and any correction in map design based on one psychophysical study alone is of limited value, especially given the incomparability between the conditions of the experiment and of real map use" (p. 161). There have been calls for more cognitive research to counter the limitations of psychophysical research in cartography (Olson 1979; Fraczek 1984), and Gilmartin (1981a) discussed the inextricable link between cognitive and psychophysical study. Peterson (1985, 1987) examined mental images of maps and comparison of pattterns on point symbol maps, rather than comparison of symbols. MacEachren's *How Maps Work* (1995) is a current, in-depth analysis of human cognitive processes used to understand map symbols and map patterns.

Beyond Circle Size Estimation

Groop and Cole (1978) studied the relative effectiveness of cut-out versus overlapping graduated circles, since many graduated circle maps include dense clusters of circles. Their analysis showed that estimation errors associated with cut-out circles were significantly greater than those associated with overlapping circles. Cut-out circles are seldom used today because of the ease of constructing overlapping circles with the computer. However, Dent (1996) noted that they "add a three-dimensional (plastic) quality to the map" (p. 177). Groop and Cole also questioned the effects of clustering on the accuracy of size estimation. Gilmartin (1981b) found that circle size estimation was affected by neighboring circles and that the effect can be minimized by including intervening linework such as boundaries separating enumeration units. Dent noted the current lack of a solution to this problem.

Researchers have examined the effects of both lightness and hue on symbol-size estimation. Meihoefer (1973) studied the effect of transparent, gray, and black circles on size estimation with his 1969 set of graded circles. He found no difference in subject perception with these three fill variations. Crawford (1971) concluded that the perception of circle size was the same whether the circles were represented in black or gray. Patton and Slocum (1985) conducted a study to assess the effect that aesthetic use of color had on pattern recall of graduated circles. Lindenberg (1986) examined the effect of color on size estimation for graduated circles. Neither Patton and Slocum nor Lindenberg found differences associated with color, substantiating Meihoefer's and Crawford's studies. In contrast, Williams (1956) found small differences in size estimates between colored symbols (circles, squares, triangles) and black symbols of the same size, with the largest difference of six percent between equivalent yellow and black symbols.

Griffin (1990) recognized the increase in cognitive study of graduated symbol use, but suggested continued research using the stimulus-response approach. He investigated visual contrast between graduated circles and their map background, user preference for opaque versus transparent circles, and the effect of varying circle fill color on size estimation. His subjects disliked a white fill most and showed a strong preference for black figures. Preference results for opacity depended on a subject's preference for clarity of the figure or detail of the background. As in Crawford (1971), Meihoefer (1973), Patton and Slocum (1985), and Lindenberg (1986), color variation was shown to have no effect on size estimation.

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Comparisons of Types of Graduated Symbols

Unfortunately, little has been written by cartographers comparing the relative usefulness of different types of graduated point symbols. This lack stems, in part, from the historical difficulty of manually constructing graduated symbols. Graduated circles were versatile and easy to construct through both manual and automated means, and have, therefore, received greater attention. In addition, few researchers have investigated the representation of comparison and proportional relationships within multivariate graduated symbols.

In 1926, Eells examined "The Relative Merits of Circles and Bars for Representing Component Parts." He noted commonly held criticisms of using circles to illustrate component parts (what we refer to as proportional relationships) including: difficulty in rapid and accurate estimation of values, inaccuracy of estimation due to their areal nature, and the suggestion that "while their popularity and psychological appeal are grudgingly admitted their use is 'an insult to man's intelligence' " (p. 122). However, Eells' study concluded that circle diagrams divided into component parts were easily and accurately read, that accuracy increased with the number of subdivisions of circles but decreased for subdivided bars, and that the use of circle diagrams was "worthy of encouragement" (p. 132). Croxton and Stryker's 1927 follow-up study supported Eells's findings and suggested that for illustrating 25, 50, and 75 percent relationships, circles worked significantly better than bars. Croxton and Stein (1932) further examined the accuracy of size estimates using bars, squares, circles, and cubes. They concluded that scaled bars yielded more accurate results compared to area or volume symbols. Furthermore, they found that performance with both circles and squares was better than with cubes. Their study supported the commonly held belief that the fewer dimensions a graduated symbol possesses, the more accurately its size is estimated (a general rule strongly supported by Cleveland, 1985, p. 254). Neither Croxton's studies nor Eells' study was performed in a cartographic context. Their results offered basic comparisons of graduated symbols without the complications of spatially registered data.

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Clarke (1959) examined the relative accuracy of size estimation for lines, circles, squares, spheres, and cubes. His data were collected using three symbol sizes in each trial. Subjects were asked to estimate the sizes of the small and large symbols using a middle-size symbol. Greater errors in estimations of size occurred as the number of dimensions increased and as the difference between the symbol and the standard increased. Ekman and Junge (1961) elaborated on this conclusion by generating power functions for symbol types.

Flannery (1971) included examination of the relative effectiveness of wedges and bars compared to circles in his research. Although he concluded that wedges were not estimated as accurately as circles, he noted their usefulness for showing proportions for specific locations (cities, ports, and intersections, for example) because the vertex naturally points to the place to which the value belongs. He also noted the relatively high accuracy of estimations with linearly scaled bars.

Crawford (1973) examined perceptions of graduated squares. Specifically, he studied the potential for linear rather than areal estimation of square size and whether squares were correctly, under-, or over-estimated. His regression model clearly indicated that square sizes were estimated areally, not linearly, and he showed that square sizes were more accurately estimated than circles. Likewise, Heino (1991) recommended the use of squares and cubes, instead of circles and spheres, for accurate estimation of data sets with large ranges.

In addition to comparing types of symbols, multivariate symbols would seem a logical topic of research in thematic cartography. Both MacEachren (1995) and Nelson and Gilmartin (1996), however, note that very little of this research has been done. One theme in their reviews of the topic is the link between the design of multivariate symbols and whether symbol dimensions representing each variable remain *separable* or are combined in a more holistic symbol with *integral* dimensions (Shortridge 1982). Nelson and Gilmartin examine symbols representing sets of four variables with attention to the multiple purposes of multivariate map symbols: from 'what' and 'how much' at the local level (which may be better represented with separable dimensions) to regional patterns and correlations between variables (which may be better represented with integral dimensions).

As an example of multivariate symbol research, Slocum (1981) examined two-sectored pie graphs. Specifically, he measured the just-noticeable difference for sector size and accuracy of sector size estimation. He found that subjects could not discriminate between sector size differences of less than nine degrees, and that sector sizes were estimated within a three-percent margin of error. He suggested rounding data to the nearest five percent, rather than one percent, before drawing sectors. His study was, in part, a response to Balogun (1978), who suggested the use of decagraphs (ten-sided polygons) for representing proportions. Examples of decagraph use can be found in the *Atlas of Newfoundland and Labrador*, where they are used to represent economic data (McManus et al. 1991, plates 10 and 19).

Point Symbol Discussion in Cartographic Textbooks

Cartography: Thematic Map Design by Dent (1996) and *Elements of Cartography* by Robinson et al. (1995) are two widely used collegiate texts. Chapter 8 of Dent's book offers a concise survey of the research that has been conducted over the past twenty-five years, including apparent scaling (Flannery 1971), effects of symbol clustering on size perception (Gilmartin 1981b), anchor effects (Cox 1976), range grading (Meihoefer 1969), and open and cut-out circles (Groop and Cole 1978). Dent also directs attention to multivariate graduated point symbols, the use of graduated squares and triangles, and the use of volumetric symbols. Robinson et al. briefly discuss the use of variations in symbol shape and orientation to represent classes of graded data. They offer an in-depth discussion of graduated circle use in representing univariate data with the addition of a color sequence for a second map variable. Their coverage of multivariate mapping is divided into four types, in their words: superimposition of features (different symbols), segmented symbols (sectored pie graphs), cross-variable mapping (bivariate choropleth), and composite indexes (cartographic modeling). They warn students that "supermaps" illustrating too many variables often hinder the clear communication intended by the mapmaker.

Issues for Future Research

Point symbol maps may be designed with multiple objectives, such as encourging accurate symbol-size estimation, easing legend matching, attracting attention, representing patterns across a map, and showing spatial relationships between variables. Success for one design objective often necessitates failure for competing objectives (Petchenik 1983), though interactive environments that encourage multiple representations improve on this discouraging reality. Further experimental testing and other investigations should be structured to account for the multiple tasks for which maps are designed.

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Examples of competing objectives can be found for varied aspects of point symbol mapping, such as range grading, symbol dimensionality, map comparison, and multivariate symbols. The literature reveals a tension between accurate estimation of individual symbols and the alternative of range-graded symbols, for which data values are classed and assigned to ordered but arbitrarily scaled symbol sizes. Range-graded symbols are selected to be obviously different in size and easily identified in a legend. A hybrid approach with symbols scaled to means of data

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classes, for example, may be a useful compromise between the comfort of easy identification and an accurate overview of relative magnitudes across a map. Similarly, low symbol dimensionality improves symbol-size estimation but may hinder map pattern interpretation. The larger range in sizes of linearly-scaled symbols (bars, for example) causes symbols to extend farther across the map than higher-dimension and more compact symbol forms (areal and volumetric symbols such as squares or cubes) representing the same data. Adjustments within a map series, such as equalizing symbol areas among maps before comparison (Peterson 1985) or using constant symbol sizes for minimum and maximum data values (Monmonier 1977), assist pattern comparison but interfere with comparison of data magnitudes between maps. Another challenge is choice of multivariate mappings with separable symbol dimensions (which may encourage comparisons within symbols and within variables) versus integral dimensions (which may improve understanding of patterns in correlations between variables). Although researching the endless variety of multivariate point symbols is a daunting prospect, it would be useful to see work on how people use and understand these types of representations.

The next sections summarize many possibilities for point symbol mapping. In addition to the theoretical research issues that are reviewed, interest in these symbols also invites practical research on how point symbols can be effectively implemented in mapping, GIS, and visualization software environments.

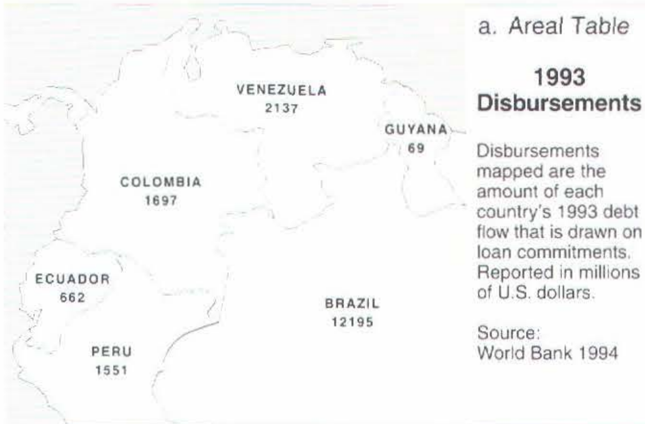
GRADUATED POINT SYMBOL EXAMPLES AND SOURCES OF INNOVATION

Figures 2 through 8 illustrate both common and innovative ways to illustrate quantitative univariate and bivariate data with graduated point symbols, and Figure 1b presents a choropleth version of the map for comparison. In addition to univariate representations, this discussion concentrates on basic ways of representing proportional and comparison relationships on bivariate maps. These terms require some explanation. 'Proportional' data refers to relationships in which one data set is part of the other data set being mapped. Similarly, Eells (1926) referred to representing proportional relationships as illustrating component parts. 'Comparison' data refers to those data that are two separate measures but the relationship between them (correlation) is of interest. One data set is not a part of the other. The data on loan disbursements that was used for the maps in this paper, for example, may be divided into public and private *proportions* or it may be *compared* to principal repayments for the same year.

Our mapped data (Table 1) were taken from a World Bank report (1994). The same data are mapped in each of the figures to aid comparison of the symbol types, though the small extent of the maps does not foster regional comparisons and does not allow evaluation of the symbols with more numerous enumeration units. All data are for countries in northern South America. Point symbols are used for country data in these maps to emphasize that *point* symbols are appropriate for *areal* data, even though some authors restrict their representations to choropleth mapping (Figure 1b). All data are for 1993 debt levels: total loan disbursements, principal repayments, and the proportions of total disbursements for public versus private loans. Table 1 notes provide further explanation of the variables.

Univariate Data: Total 1993 Disbursements

Figures 1 through 4 illustrate a variety of methods for mapping univariate data. Figure 1a lists disbursement values as an areal table. Figure 1b is a choropleth map of these data, which is shown for compari-



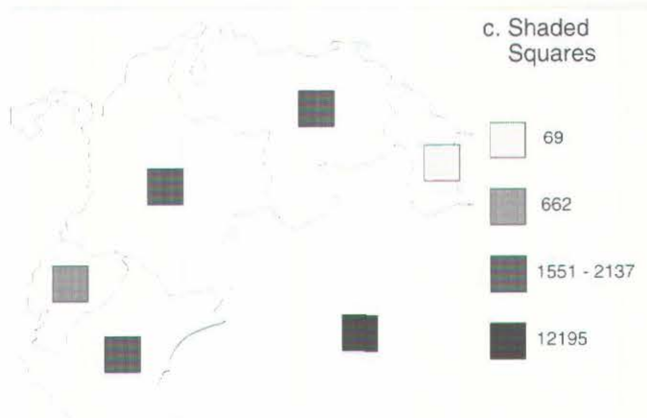


Figure 1. Areal table (a), choropleth (b), and shaded square (c) representations of loan disbursement data. The many point symbols described in this paper are alternatives to the flawed representations in a and b. Numbers (a) do not provide a visual representation of relative magnitudes of data values. The sizes of the gray fills of the choropleth map (b) are controlled by the sizes of the countries rather than the data values. Country size has a large visual impact on the data representation but it is not directly related to the economic variable of interest. For this reason, cartographers generally admonish against representing total values with a choropleth map. Shaded point symbols of a constant size, such as squares (c), are one way of removing the effect of enumeration-unit size on a gray-scale symbolization.

Country	Loan Disbursements[a]	Principal Repayments[b]	Disbursements to Public Entities[c]
Brazil	12195	6212	3265
Colombia	1697	2083	1547
Ecuador	662	488	497
Guyana	69	46	69
Peru	1551	1007	1492
Venezuela	2137	1515	1877

Table 1. Debt Data for Maps in All Figures

Notes:

All data are for 1993 and are reported in millions of U.S. dollars. The data source is a 1994 World Bank publication titled *World Debt Tables, External Finance for Developing Countries,Volume 2.*

[a] Loan disbursements are defined as "drawings on loan commitments during the year specified" (p. *xv*). Figures 1 through 4 show univariate maps of disbursements.

[b] Principal repayments are "the amounts of principal (amortization) paid in foreign currency, goods, or services in the year specified" (p. *xv*). Principal repayments are compared to loan disbursements in Figures 6 and 7a.

[c] Disbursements to public entities are external obligations of public debtors, "including the national government, a political subdivision (or an agency of either), and autonomous public bodies" (p. *xv*). The remainder of loans are to private debtors who are not guaranteed by public entities. Public and private disbursements are the proportions of total disbursements mapped in Figures 7b and 8.

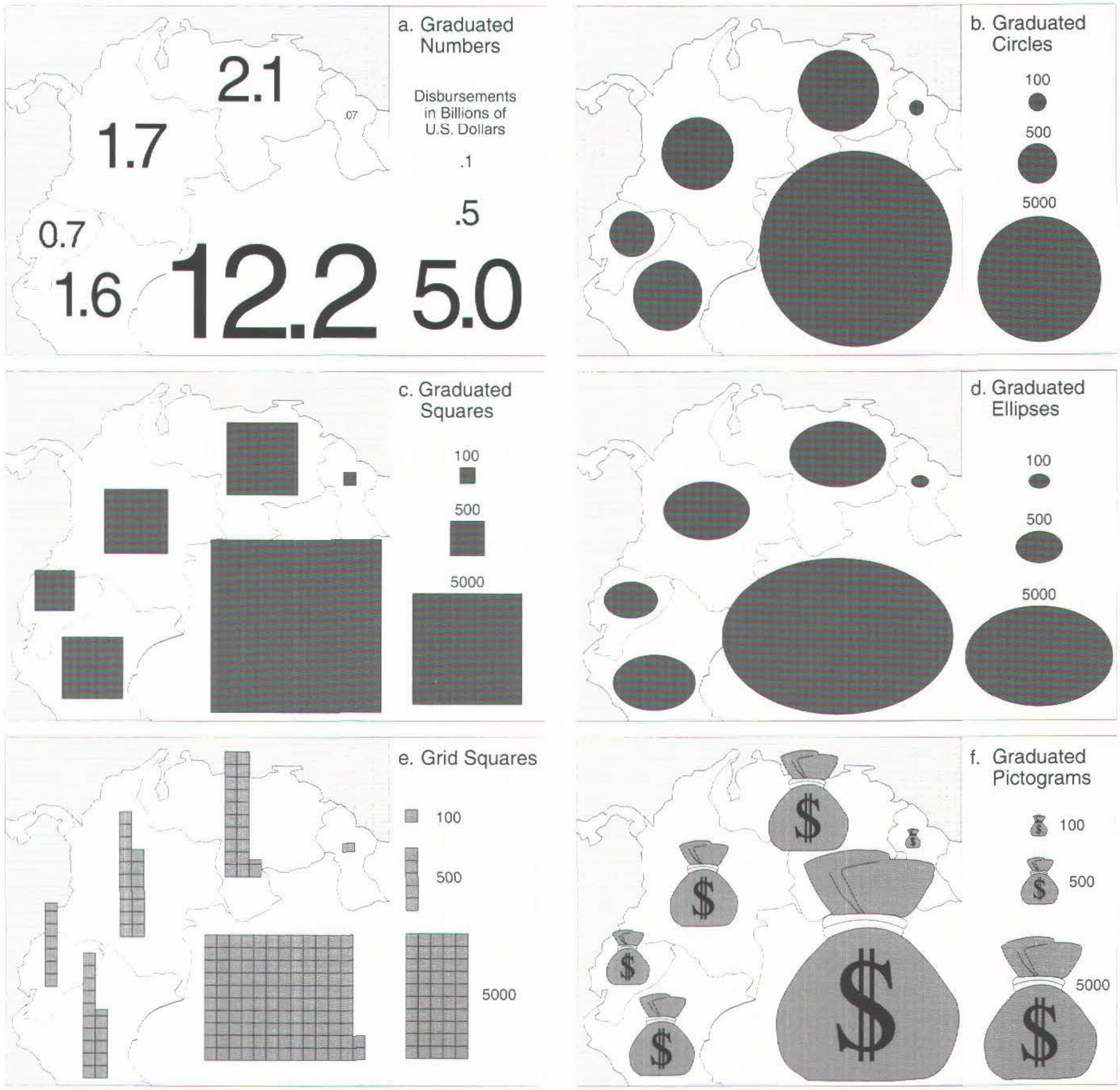


Figure 2. Areal univariate point symbols representing 1993 loan disbursement data: graduated numbers (a), graduated circles (b), graduated squares (c), graduated ellipses (d), grid squares (e), and graduated pictograms (f). Data values are in millions of U.S. dollars, except for Figure 2a which shows billions of U.S. dollars (see Table 1 for description of mapped data). Symbols are sized to data values using square-root

son but is not a recommended representation for these total dollar values. Figure 1c is a variation on the gray-scale representation of the choropleth map, with the gray fills applied to a repeated square symbol of constant size. This variation removes the unwanted effects of variation in enumeration-unit size, though differences in lightness are more difficult to compare than differences in symbol area (Cleveland 1985, p. 254). Pazner (1997) proposed a similar representation with constant-size squares that he termed "tile maps." Size and lightness may also be used as redundant symbolizations (each representing the same data values within symbols), and lightness may be used to map a second variable within a graduated

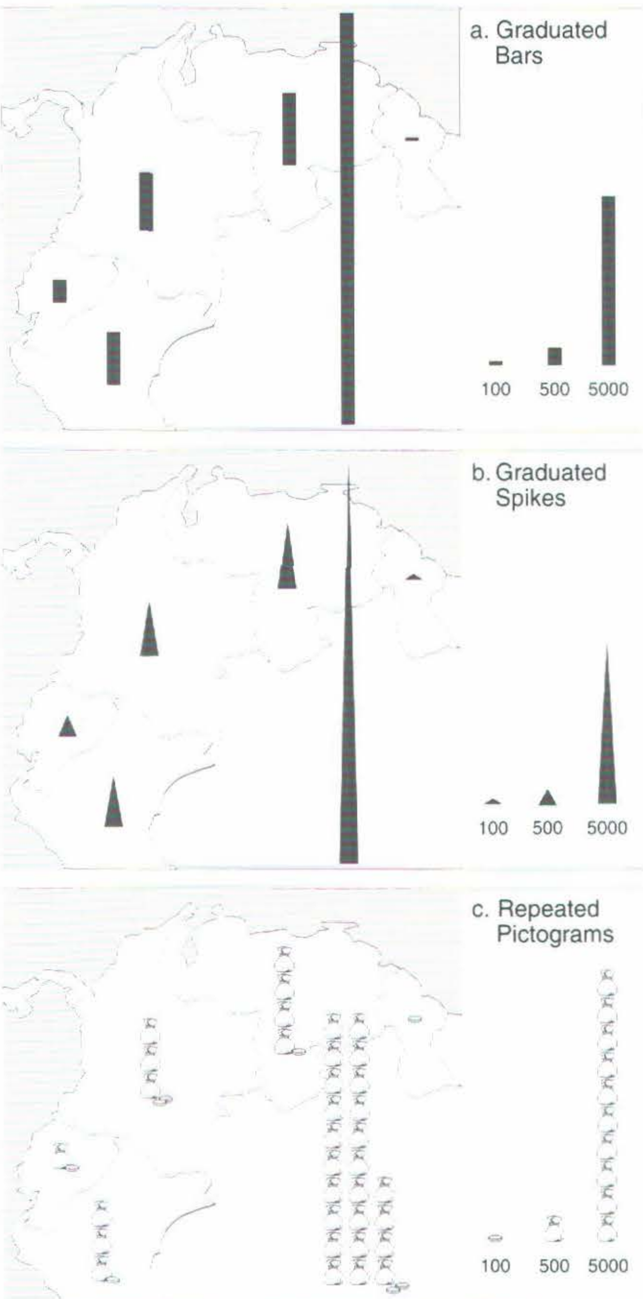


Figure 3. Linear univariate point symbols representing 1993 loan disbursement data: graduated bars (a); graduated spikes, formed by triangles with a constant base and heights linearly scaled to data (b); and stacks of repeated pictograms (c).

symbol. These valid variations are not pursued because this paper has been limited to a manageable set of possibilities by focusing primarily on graduated symbol forms.

Like Figure 1a, Figure 2a is an areal table with the sizes of numbers graduated by their values. Graduated numbers offer the reader the accuracy of a table with the advantage of a visual representation of relative magnitudes. (Note that Figure 2a is the only map with symbols that represent debt in billions of dollars; Figure 1a and all other map legends list debt in millions of dollars.)

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Traditional graduated circles and squares also work well with the disbursements data set (Figure 2b and c). Their familiarity and versatility make them mapping options that should not be overlooked. By sizing the height and width of the entire set of symbols by a constant ratio, graduated ellipses (Figure 2d) offer a mapping solution for sets of enumeration

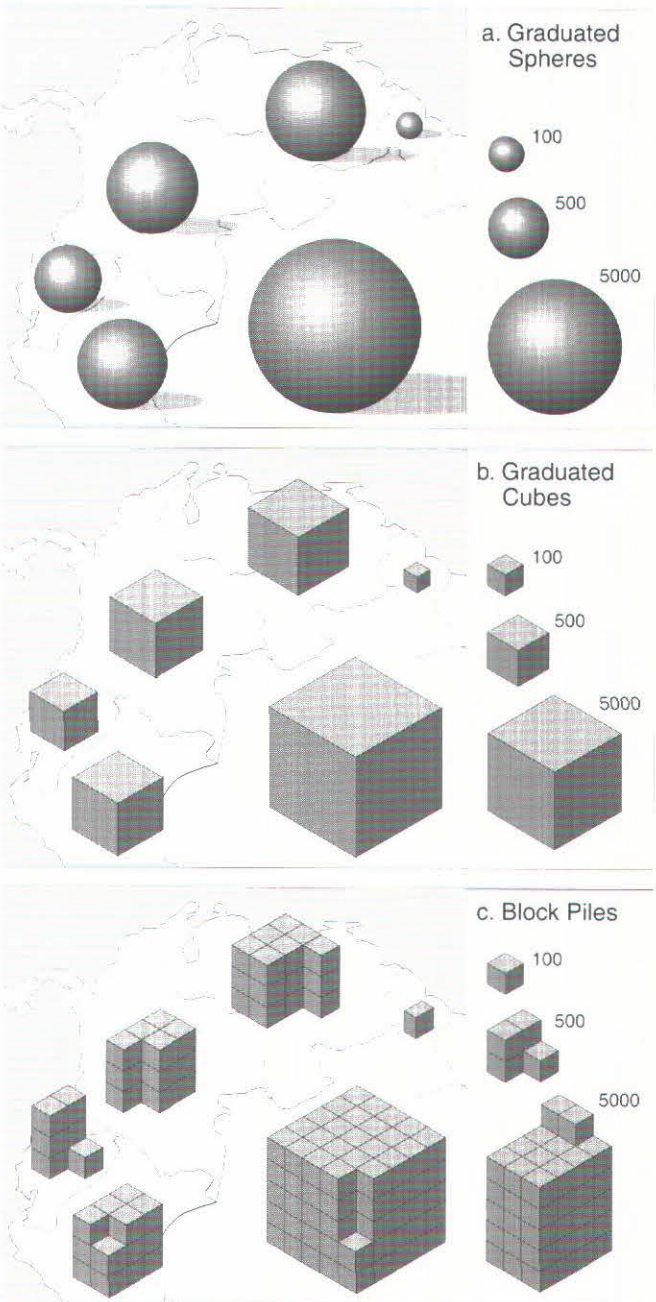


Figure 4. Volumetric univariate point symbols representing 1993 loan disbursement data: graduated spheres (a), graduated cubes (b), and block piles (c). Cube-root scaling is used to size the spheres and cubes, and block piles are constructed by repeating the smallest cube symbol.

units having generally elongated shapes. These ellipses are also included for comparison to a later bivariate use (Figure 6g). Magnitudes represented with a square grid (Figure 2e) may aid accurate interpretation because individual cells can be counted.

Another areal symbol, the 'money bag' pictogram (Figure 2f) offers an example shape that more literally reflects the data, captures the reader's attention, and may be scaled to represent relative quantities. Pictograms are particularly well suited when 'catching' the reader's eye is a priority, making them common in magazines, newspapers, and educational material. *The New State of the World Atlas* (Kidron and Segal 1984) offers good examples of the unlimited possibilities of pictogram use. This atlas and later editions (for example, the fifth edition was published in 1995) use graded pictograms to cleverly illustrate everything from radio receivers per 1000 people to worker exploitation rates. An example similar to that used in Figure 2f can be found in the *Atlas of Newfoundland and Labrador* (McManus et al. 1991, pl. 19). Although pictograms may not be well-suited for accurate magnitude estimation, variety and ability to match the theme of the data being mapped make them a worthwhile consideration.

Linearly-scaled symbols (Figure 3) work well for these data because of the relatively large enumeration units. The bars in Figure 3a overlap multiple enumeration units and may be difficult to associate with the correct location. In contrast, the larger bases of the spikes in Figure 3b better associate the symbols with their enumeration units. Stacks of repeated pictograms (Figure 3c) offer a linearly-scaled version of pictograms (a similar pictogram is areally scaled in Figure 2f). Alternatively, scaling the height of a single tall pictogram for each enumeration unit is also appropriate. For mapping phenomena such as mercury in sediments, makers of *The National Atlas of Sweden: The Environment* (Bernes and Grundsten 1992) use linearly-scaled graduated columns set on an oblique perspective projection of Sweden viewed from the east. This technique produces an interesting map and allows the uncluttered use of linearly scaled bars that would be stacked atop one another in a more traditional planimetric view oriented to the north.

Finally, volumetric symbols such as cubes and spheres (Figures 4a and b) are used for data with an extreme range and are relatively easy to construct. Spheres can be rendered using a circle with a radial fill and a shadow (Figure 4a). Cube construction is more challenging because of projection options (Mackay 1953; Ekman et al.

1961; Brewer 1982; Dubery and Willats 1983; Figure 5). Block piles (Figure 4c) have long been associated with Raisz's talent for cartographic symbolization (1939, 1948).

Comparison Data: Disbursements and Principal Repayments

The possibilities for comparing data with point symbols on bivariate maps are limitless. The focus will be on several combinations of circles and squares, as well as bars, ellipses, and cubes. The large number of arrangement options available using common graduated circles and squares (side by side or overlayed) offer different visual impressions while illustrating the same data (Figure 6a, b, d, and f). Overlapping the symbols produces a more integral symbol that involves pattern recognition because differences in relative values produce different overall symbol forms (compare the mostly black symbol for Colombia, which is the only country for which payments exceed disbursements, to the others). Combining shapes for two qualitatively different variables, such as graduated circles and squares (Figure 6f), increases the variation in overall symbol forms but makes relative sizes difficult to compare.

There are many options beyond combinations of circles and squares for comparing data. Adjacent semicircles (Figure 6c) clearly illustrate differences between the two data sets. However, the adjacent straight edges may lead to linear rather than area-based estimates of value differences. Estimates of differences between pairs of linear symbols are easier to make, although graduated bars (Figure 6e) can be more 'cumbersome' because linear scaling increases the range of the map covered by symbols (square-root scaling for areal symbols and cube-root scaling for volumetric symbols produce more compact shapes). One student (Jeff Erickson) offered a creative combination for pairing graduated cubes (Figure 7a). Cubes may also be placed side by side or stacked. An example of using pictograms to illustrate comparison data can be found in Thomas's *Third World Atlas* (1994). He used a split textbook to illustrate "Gross primary school enrollment ratio" (p. 59) between males and females. The left side of the book was linearly scaled to the percentage of male enrollment; the right side to female enrollment.

Ellipses (Figure 6g) may be constructed with their axes scaled to illustrate different sets of data, in this case disbursement on the horizontal axis and principal repayments on the vertical. MacEachren (1995, p. 90) describes this combination as an integration of attributes merged into one symbol. He cites Garner (1976) when stating "that object integration is more likely to lead to integral or configural conjunction than will two distinct spatially contiguous objects (e.g., paired bars on a bar chart)" (p. 90). The axes of these ellipses were scaled using the linear dimensions of the bars in Figure 6e. When ellipses are highly elongated in one direction or the other, the reader will recognize spatial patterns in variable differences by the orientation of the symbols' major axes. Ellipses with more circular shapes represent comparisons for which neither of the mapped values are significantly larger than the other. Rase (1987) included bivariate ellipses in his point-symbol mapping software.

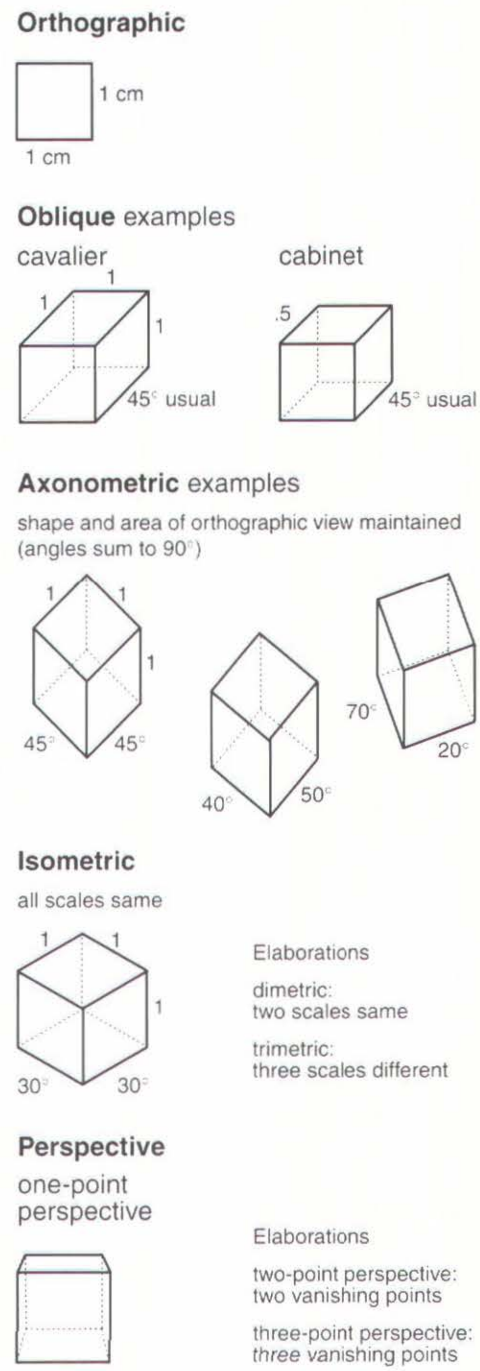
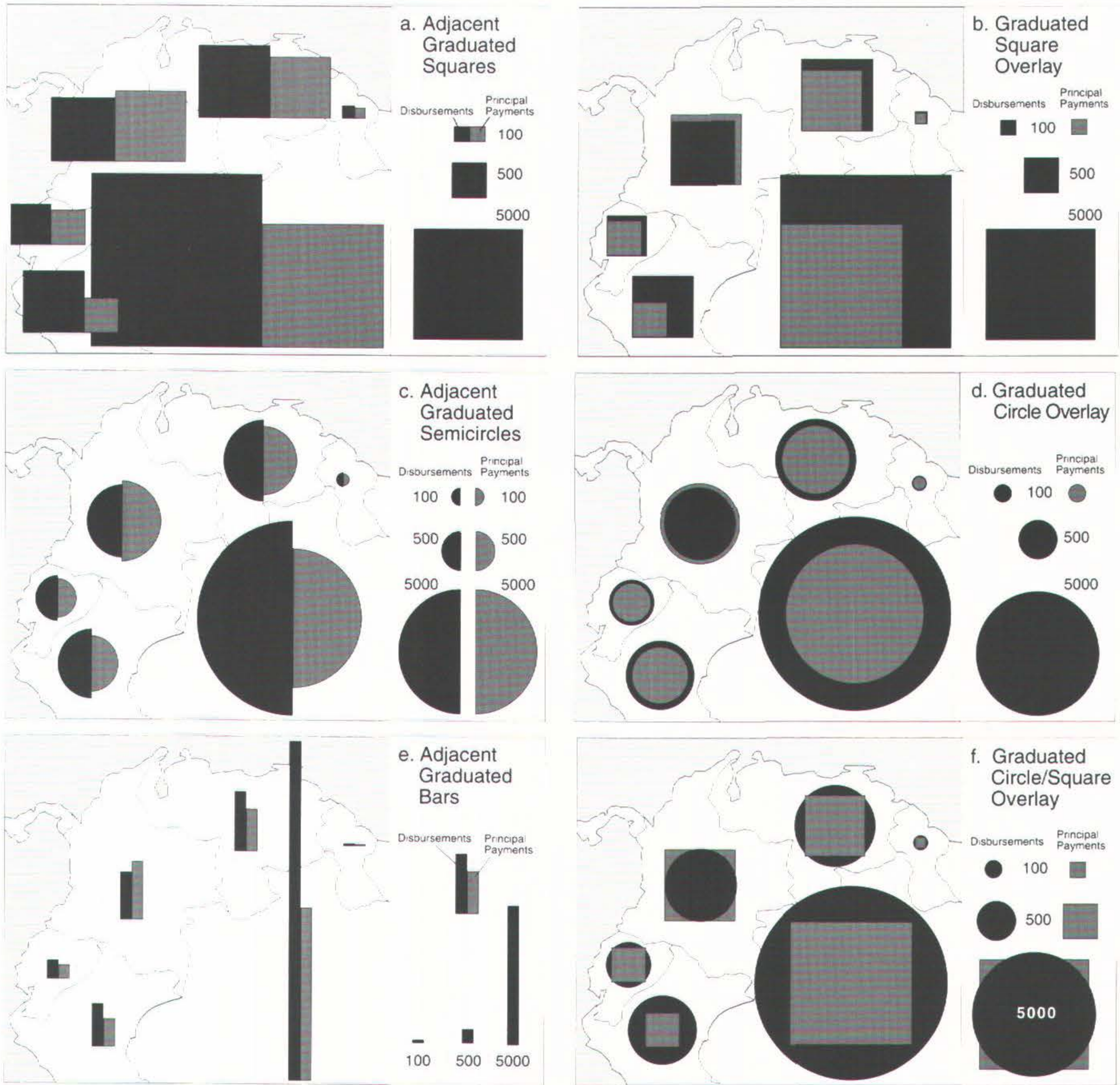


Figure 5. A sampling of the many ways of drawing three-dimensional symbols, using a 1-cm cube to demonstrate the effects of various projection systems (Dubery and Willats 1983). The cube projections generally increase in sophistication from the top down in this figure.



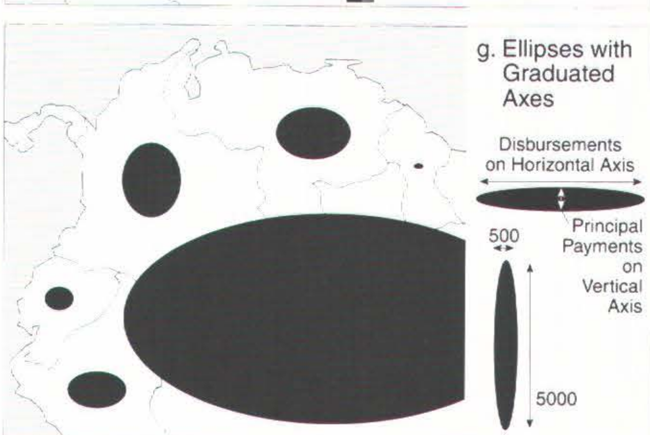


Figure 6. Areal and linear bivariate point symbols for data comparison. Related variables, 1993 disbursements and principal repayments, are compared on each map. Note that payments exceed disbursements for Colombia. Thus, a representation of repayments as a proportion of disbursements (a circle segment for example) would be quite awkward from both practical and logical perspectives. Example comparison symbols are: adjacent graduated squares (a), graduated square overlay (b), adjacent (areally scaled) graduated semicircles (c), graduated circle overlay (d), adjacent graduated bars (e), graduated circle and square overlay (f), and ellipses with graduated axes (g). Ellipse axes in g are linearly scaled to two data values to produce both varied shapes and sizes, though symbol areas are not directly scaled to a mapped variable. The overall look of the overlay symbols (b, d, and f) change markedly depending on which variable is the larger of the two. Overlay symbol construction is awkward where amounts are near equal because symbols are almost the same size but the slightly smaller one will take visual precedence (note Guyana's gray symbol).

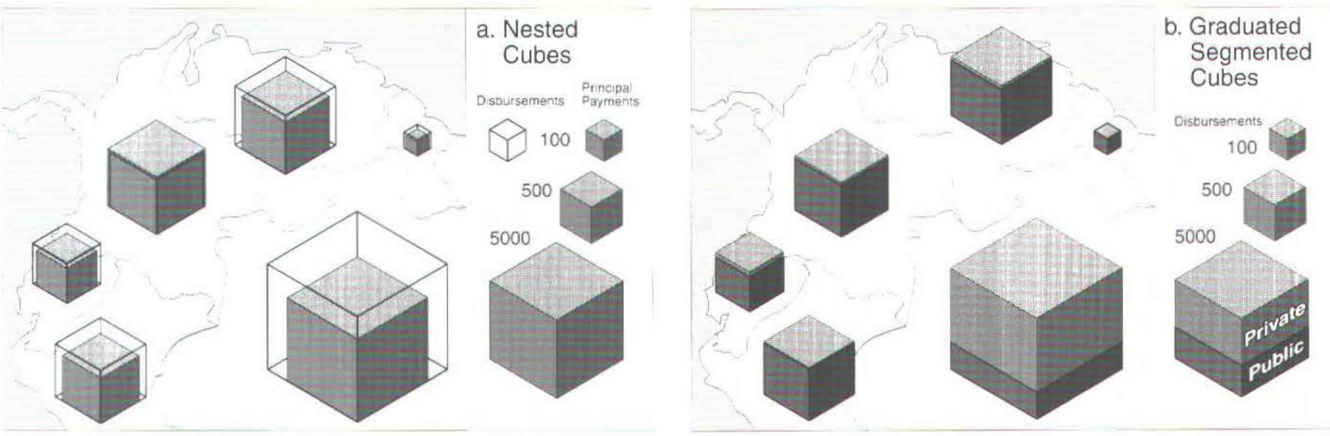


Figure 7. Volumetric bivariate point symbols for both comparison (a) and proportional (b) relationships. The same data represented in Figures 6 and 8 are used for the cubes in 7a and b respectively.

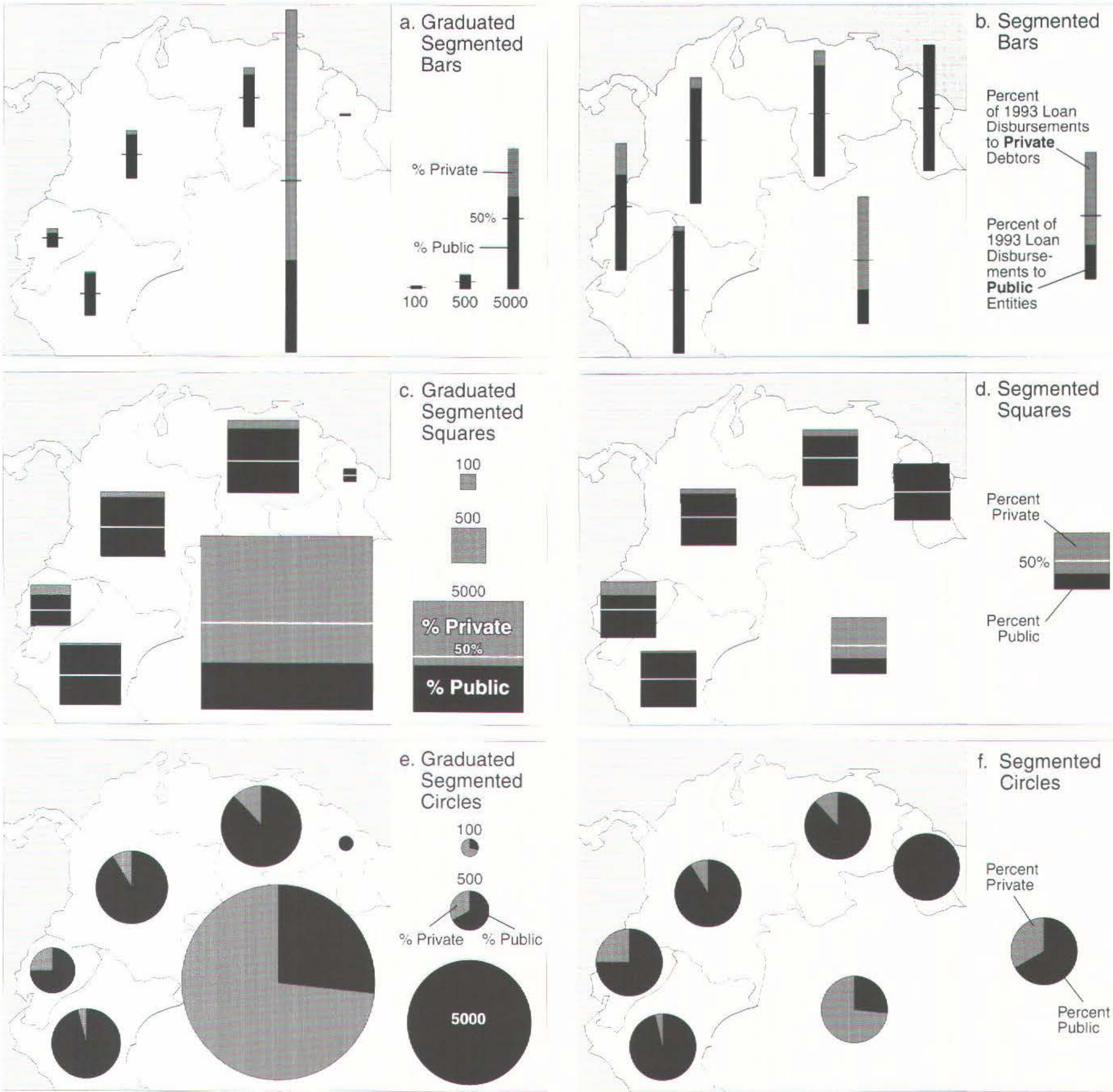
Proportional Data: Public and Private Components of Disbursements

Mapping proportional data differs substantially from mapping comparison data because the smaller of the two data sets is always part of the larger set, unlike the disbursement and repayment data used in the comparison examples (Figure 6 and 7a). In Figures 7b and 8a, c, e, and g, graduated symbols are scaled to represent total disbursements and are segmented by the proportions of loans to public and private entities. Symbols of constant size in Figures 8b, d, and f are segmented to represent only the proportions of the disbursements' compositions, not the total disbursement value (as the graduated symbols do) so they are akin to univariate symbols. In all cases, the black portions of each symbol represent disbursements to public entities and the gray portions to private. For example, the use of graduated segmented circles (Figure 8e) allows the map reader to compare the total value of disbursements among countries as well as the proportions of private and public disbursements. In contrast, maintaining a constant symbol size (Figure 8f) focuses the reader's attention on the proportional relationship.

"In contrast, maintaining a constant symbol size (Figure 8f) focuses the reader's attention on the proportional relationship."

Examples of sectored circles and wedges (Fig 8e, f, and g) are found in a variety of sources including *Historical Atlas of Canada, Vol. III* (Kerr and Holdsworth 1990), *The National Atlas of Sweden: The Environment* (Bernes and Grundsten 1992), and *The Maritime Provinces Atlas* (McCalla 1988). The *Historical Atlas of Canada* makes extensive use of sectored circles, and often uses the space surrounding the map to allow extreme symbol sizing. Wedges, often used in *The National Atlas of Sweden*, are particularly well suited for illustrating quantitative data associated with points like ports or cities (p. 97 for example). Wedges can easily be rotated to fit in where other 'bulky' symbols can not easily be associated with a point location. The graduated wedge radius represents the total symbol size without showing the entire circle from which the segment came (Figure 8g). The wedge's point anchors it to its enumeration unit (the opposite association is used for graduated spikes; Figure 3b). *The Maritime Provinces Atlas* uses a grouping of four wedges set at ninety degrees, like four flower petals. Each wedge represents the amount of a particular type of overnight accommodation across the region. The reader may either compare entire symbol structures or individual segments of each symbol.

The segmented bars and squares were constructed with fifty-percent markers (Figure 8a, b, c, and d). Students often included these reference



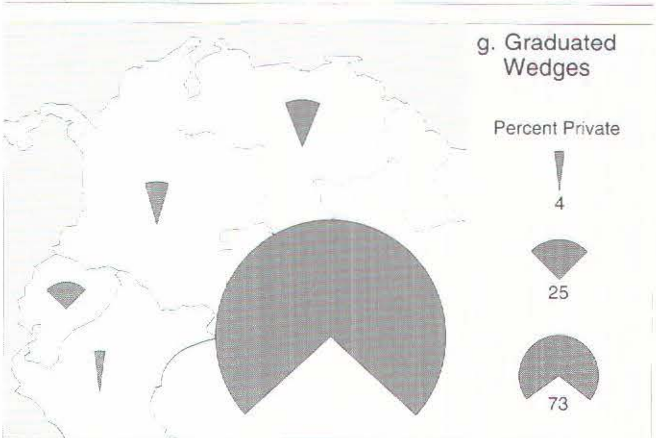


Figure 8. Linear and areal bivariate point symbols for a proportional data relationship: public and private disbursements are proportions of total 1993 disbursements. Example proportion representations are: graduated segmented bars (a), segmented bars of constant size (b), graduated segmented squares (c), segmented squares (d), graduated segmented circles (e), segmented circles (f), and graduated wedges (g). Figures b, d, and f show only the proportions of private and public disbursements and not the total amounts of disbursements, which are shown in a, c, and e. Figure g shows only the portion of private disbursements as circle sectors, or wedges, with an indication of the total-disbursement graduated circles in the square-root scaled sector radii.

points in their symbol designs, which assists accurate percentage estimates. The symbol form in Figure 8b is known to statisticians as a framed rectangle (Cleveland 1985, p. 208; Dunn 1987, 1988), and Monmonier (1993, p. 65) describes its usefulness in representing both absolute magnitudes and intensities. Dunn (1987) also suggests scaling the widths of the rectangles to total data values (to total disbursements for Figure 8b). In a conversation about Figure 8, MacEachren mentioned a successful student project from an ACSM design competition that divided graduated squares into ten-by-ten grids such that each cell represented one percent, and each row ten percent, of the whole for all symbol sizes (rather than using grid cells of a fixed size; a hybrid of Figure 8c and 2e).

CONCLUSION

The work of students new to cartography has been an inspiration. With modern computing at their disposal and freedom from the conventions of traditional thematic mapping, students are producing a wide variety of creative symbols for mapping data usually symbolized by graduated circles or squares. For example, a recent student assignment with economic data produced creative ideas ranging from a proportionally scaled Monopoly game character (with pockets turned out or clutching money bags) to weight scales drawn tipped to illustrate balances between dollar amounts (by Elliott Westerman and Erika Bozza respectively). Are these effective symbols? With so little research into multivariate representations, one can not say, but this type of experimentation can be encouraged until there is evidence to the contrary.

". . . there are wide-ranging possibilities for applying creative, eye-catching symbol designs to summarize and synthesize quantitative spatial distributions."

The intent of this paper has been to review some of the major topics on which graduated symbol research has been conducted and to demonstrate the wide variety of ways to map univariate data as well as bivariate comparison and proportional relationships. The survey was not exhaustive, but if it has inspired recall of other symbol forms that have been missed, then it has been successful in provoking consideration of the wide range of possibilities available with point symbols. Modern computer mapping allows mapmakers greater flexibility in designing creative graduated point symbols. This increase in flexibility increases the importance of research examining symbol design issues. As this article has illustrated, there are wide-ranging possibilities for applying creative, eye-catching symbol designs to summarize and synthesize quantitative spatial distributions.

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