Hillshading With Oriented Halftones

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Hillshading renders a surface with a three-dimensional appearance using shades of gray. Although shades appear as continuous tones, they must undergo a halftoning process for use with most computer output devices. This process generally uses one pattern of black and white pixels for each shade of gray, while attempting to make patterns associated with black and white pixels as difficult to detect as possible.

The method described in this paper adds aspect information to hillshaded maps with oriented halftones. Twelve orientations of clustered-dot ordered dithers represent 30° intervals of aspect. Additionally, dithering matches sixteen shades of gray associated with analytical hillshading, with each interval representing 16 of 256 shades of gray. This process allows pattern and gray tone representations of the surface simultaneously.

INTRODUCTION

Hillshading is a cartographic technique that has been used to represent continuous surfaces with shades of gray for hundreds of years (Imhof, 1982; Horn, 1981; Horn, 1982; Robinson *et al*. 1995). The tonal variations on the resulting map give the impression of a three-dimensional surface (Clarke, 1995; Horn, 1981; Horn, 1982). This effect is based on chiaroscuro, the interplay of light and shading commonly used by Renaissance artists (Burrough and McDonnell, 1998; Horn, 1981; Horn, 1982). To achieve this effect, the cartographer selects a direction of illumination and makes surface elements most directly illuminated bright and those more obliquely illuminated increasingly dark. Such hillshading uses the Lambertian assumption of an ideal diffuse reflector approximated by a matte surface. A brightness value is calculated for each surface element based on the cosine of the angle θ between the selected illumination vector and the normal vector for each surface element.

Historically, cartographers working with hillshading focused on the methods of producing this effect, but also needed to concern themselves with the technologies for reproducing the maps. The hachure method devised by Lehmann (1799) used fine lines drawn in the direction of steepest topographic gradient. It was widely used because copper engraving, a popular reproduction technique for maps at the time, could not reproduce continuous shades of gray, but worked well for black lines on white paper to approximate grayscales. Only after the patent of the halftone screen in 1865 and the crossline screen in 1869, did lithographs that allowed the continuous variations of shades of gray become a commercial success (Horn, 1981; Horn, 1982).

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Computer cartographers working with hillshading also necessarily concerned themselves with reproduction methods. Before plotters could output continuous shades of gray, researchers automated two methods devised by Tanaka (1932 and 1950) that used contour lines to approximate the hillshading effect. Peucker *et al*. (1972) were the first to automate these methods. An alternative algorithm for Tanaka's orthographic relief method was later suggested by Yoeli (1976). Brassel *et al*. (1974) devised a

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method for creating the hillshading effect with a special character set on a line printer. Finally computer cartographers were able to reproduce hill-shading maps with continuous shades of gray with new gray tone plotters (Peucker *et al*., 1974).

"Printed maps with hillshading are not continuous tones of gray, but are composed of black halftones on white paper that give this illusion."

With the ease of computer-based hillshading methods and the availability of continuous tone plotters, hillshaded maps have been produced and reproduced widely. From a cartographic perspective, current hill-shading methods are the solution to surface rendering with the Lambertian assumption. From a computer graphics perspective, however, part of the reproduction method could be exploited to add information to the map. Printed maps with hillshading are not continuous tones of gray, but are composed of black halftones on white paper that give this illusion. The method developed and discussed here assigns different orientations to halftone patterns based on the aspect of the surface, matches shades of gray based on the hillshading values, and combines pattern and shading in a single hillshaded map.

PREVIOUS CARTOGRAPHIC RESEARCH

Variations in tone have been used to create the hillshading effect greater than or the same resolution as the data being mapped. Hillshading greater than the resolution of the data frequently used black point symbols on a white background, while hillshading at the resolution of the data frequently used continuous shades of gray.

Eckert (1921) used dots of various sizes to create a hillshaded map of the Lake Lucerne region of Switzerland (see **Figure 1**). With his use of

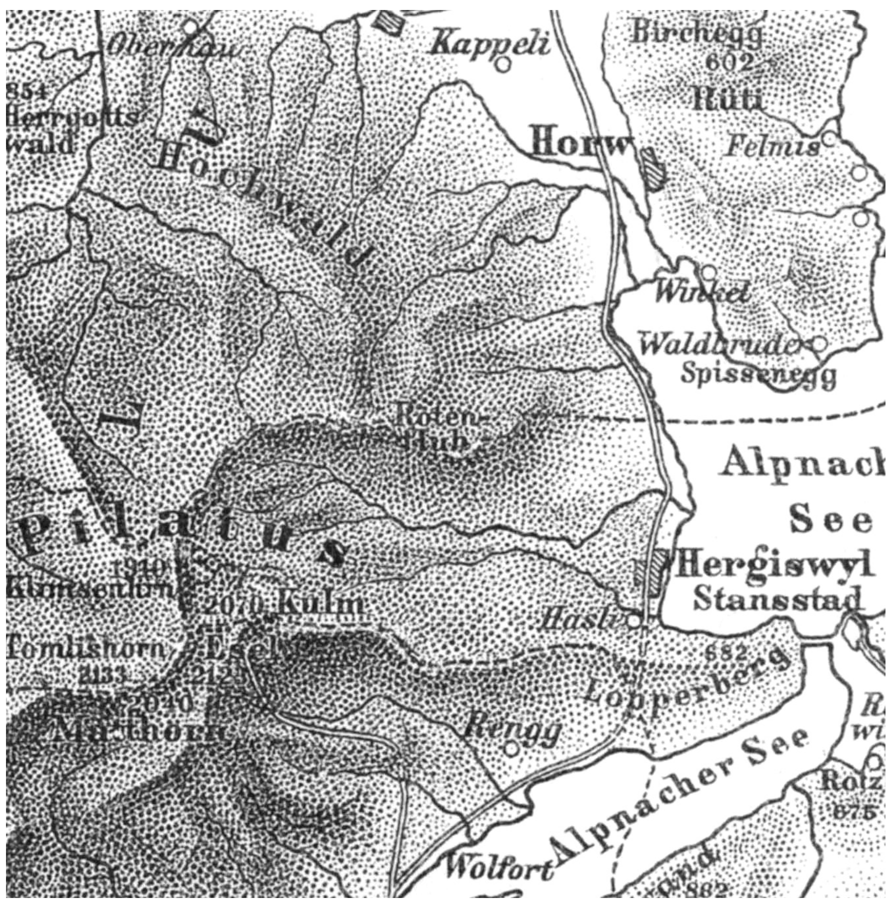

Figure 1. A portion of Max Eckert's dot map of the Lake Lucerne region of Switzerland. (From Eckert (1921) with permission from Walter de Gruyter publishers, Berlin)

large and small dots, the landscape has the appearance of an enlargement of a halftone image printed by press. These macroscopic dots proved difficult to draw, overloaded the map, and never found popular use (Imhof, 1982).

Vertical hachures were used widely in the first half of the 19th century. Lehmann (1799) quantified a method by which large-scale hachures could be drawn in the direction of slope, with the thickness of the hachure being proportional to the steepness of the slope. The hachure method with an oblique illumination effect reached its cartographic and artistic apex with the Swiss "Dufour" maps (see **Figure 2**). The first editions of these 25 hachure maps were published at 1:100,000 by the Swiss Federal Office of Topography between 1845 and 1865 (See http://www.swisstopo.ch/en/maps/ak/tk.htm for more detail).

The process was computer-automated by Yoeli (1985) (see **Figure 3**). Both manual and computer hachure methods were based on a contour framework. Although relating hachures to the resolution of the data is difficult, it would be fair to say that the hachures were less detailed than the contours from which they were created. Kennelly and Kimerling (2000) automated a method for creating small-scale hachure maps based on regularly spaced points instead of a contour framework (see **Figure 4**). These hachures are well above the resolution of the data, with Digital Elevation Model (DEM) grid cells first aggregated into 3x3 grid cells, then each new grid cell represented by a single hachure.

Horizontal hachures can also create tonal variations associated with hillshading (Imhof, 1982). Instead of lines in the direction of maximum slope, horizontal hachures are lines in the direction of no change in elevation (see **Figure 5**). These horizontal form lines are similar to contours,

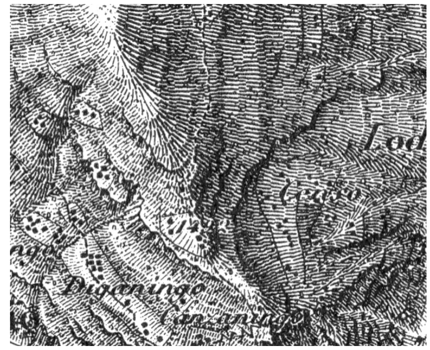

Figure 2. A portion of a large-scale topographic map using hachures to represent topography. (From Sheet 19 [1858], Switzerland, 1:100,000, the "Dufour map.")

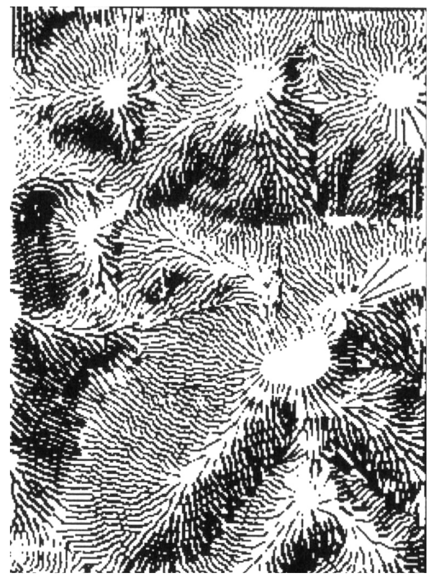

Figure 3. A computer-automated large-scale topographic map south of Haifa, Israel using hachures to represent topography (From Yoeli (1985) with permission from the Cartographic Journal*).*

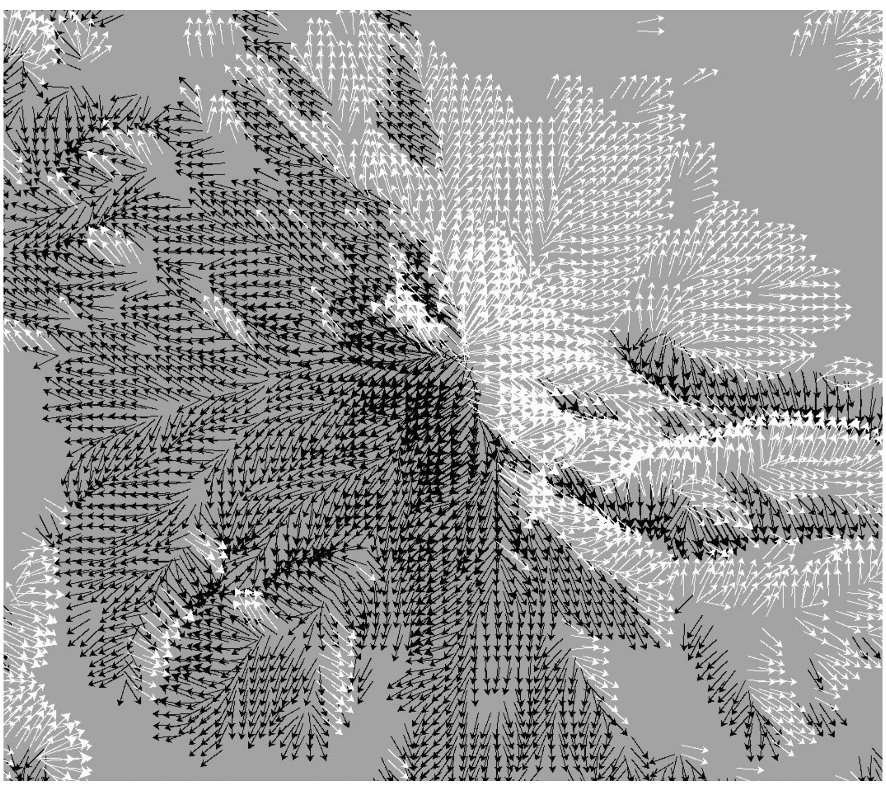

Figure 4. A computer-automated small-scale topographic map of Mt. Adams, Washington using hachures to represent topography (From Kennelly and Kimerling (2000) with permission from Cartographic Perspectives*).*

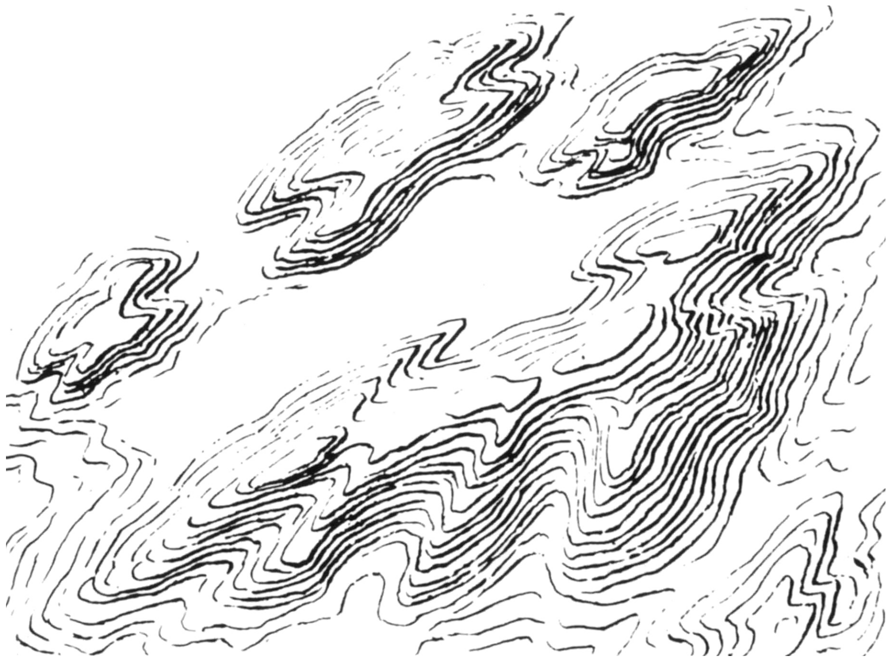

Figure 5. An example of horizontal hachures (From Imhof (1982) with permission from Walter de Gruyter publishers, Berlin).

but more compactly and evenly arranged, not assigned an exact elevation value, and vary in thickness based on hillshading values. Horizontal hachures were often used in the 19th century, but are not commonly used today.

Hillshading with tonal variations at the resolution of the data includes both manual and computer automated methods. Manual hillshading, like hachuring, often was based on a contour framework, with shades of gray interpreted at and between contour lines (Imhof, 1982; Yoeli, 1959). This method created striking portrayals of topography when used by skilled cartographers (Imhof, 1982).

"Patterns (e.g. hachures, contours) have been overprinted on hillshading, but cartographers have never attempted to use hillshading itself to create patterns."

Computer-based hillshading often operates on a DEM, with the angle and resulting shade of gray calculated for each grid cell. Although a straightforward process (Peucker *et al*., 1974), the earliest development of the method was nearly thwarted due to technological constraints. Yoeli (1965) used an early computer technology to calculate hillshading values for a 25x25 topography grid, but had no way of outputting his results without a gray tone plotter. Undaunted, he screened a gray wedge of ten classes, cut the screen into 2 cm. squares, and created a mosaic based on the computed hillshading value of each grid cell. Next, Yoeli (1966) related the light intensity calculated for hillshading with the density of black dots a white background. Finally, Yoeli (1967) was able to use a computer-controlled electronic typewriter to print and overprint desired characters into each cell of a hillshaded grid (see **Figure 6**). In this manner, he could match 20 shades of gray from dot densities to the brightness values associated with hillshading.

Inherent differences stand out between methods greater than or the same resolution as the data. Hachuring results in strong patterns associated with the form of the topography. Hachuring, however, results in a map that only approximates values calculated for hillshading with the Lambertian assumption. Hillshading results in smooth variations in shades of gray. Patterns (e.g. hachures, contours) have been overprinted on hillshading, but cartographers have never attempted to use hillshading itself to

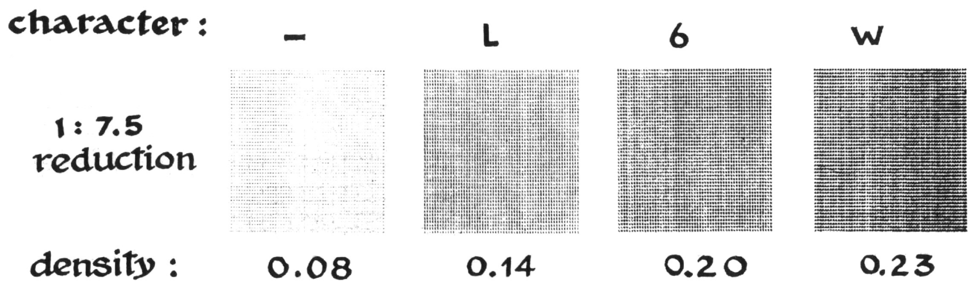

Figure 6. Examples of various densities of gray represented by repetitive printing of a single character within a grid cell (From Yoeli (1967) with permission from the Cartographic Journal*).*

create patterns. This project describes how this can be implemented with digital halftones.

COMPUTER GRAPHICS AND HALFTONES

In the field of computer graphics, shading or rendering includes illumination models for diffuse reflectors that are identical to cartographic hillshading techniques using the Lambertian assumption (Shirley, 2002; Hearn and Baker, 1994; Zhou, 1992; Foley *et al*., 1990; Rogers, 1985). Cartographers and computer graphic professionals, however, often render different sorts of data. Cartographers render detailed elevation data, which reveals small scale geometry and large scale detail of the landscape. Computer graphic professionals often use geometric shapes approximated with polygonal surfaces (Shirley, 2002; Hearn and Baker, 1994; Foley *et al*., 1990). Resulting edges at surface boundaries are smoothed using a number of interpolated shading methods, such as Gouraud shading. (Hearn and Baker, 1994; Foley *et al*., 1990; Rogers, 1985). Cartographers also have used Gouraud shading to smooth edges of topographic models (Weibel and Heller, 1991).

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Surface details often are added to computer-rendered graphics, as "smooth, uniform surfaces [appear] in marked contrast to most of the surfaces we see and feel." (Foley *et al*., 1990, 741). Two general methods are used to map texture onto a smooth surface, one a mathematical mapping function and the other a perturbation function (Rogers, 1985). Two dimensional texture mapping uses a coordinate transformation to map a pattern or image onto a surface (Shirley, 2002; Foley *et al*., 1990). The resulting graphic continues to appear geometrically smooth and may not follow patterns associated with principles of shading.

Bump mapping overcomes this problem by adding irregularities to the surface before shading. Foley *et al*. (1990) explain; "A bump map is an array of displacements, each of which can be used to displacing a point on a surface a little above or below that point's actual position" (744). In practice, bump mapping perturbs the surface normal vector before shading is calculated using mathematical functions, including ones that are random or periodic (Hearn and Baker, 1994). In this manner, correctly shaded texture can be added to a surface.

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Patterns can be added to smooth surfaces with texture and bump mapping, but no new information is added from the original surfaces. Patterns based on data can be displayed on rough topographic surfaces, adding new information from the original surface. One method for adding texture is through the use of derivatives of the DEM, such as fractals (e.g. Clarke, 1988). McCullagh (1998) points out one issue with using a regional statistical value on individual cells; "The roughness introduced into the landscape may be statistically correct and visually effective but will not be locationally accurate" (102).

The method used in this project addresses this concern, by basing texture on local derivatives of the DEM, which vary at each grid cell. The local variable is the aspect direction, the same variable used in hachure maps. The detailed texture is created with digital halftones.

"Halftoning may use ordered or random dithers, with ordered dithers allowing control over the dithering pattern."

Ulichney (1987) states "Digital halftoning, also referred to as spatial dithering, is the method of rendering the illusion of continuous-tone pictures on displays that are capable of producing only binary picture elements." (xiii). Halftoning may be necessary for video displays that are implemented with frame buffers that limit gray scale capability, and are almost always necessary for raster images with computer hard copy devices (Ulichney, 1987).

Halftoning may use ordered or random dithers, with ordered dithers allowing control over the dithering pattern. Ordered dithers can be clustered or dispersed. Clustered dithers are used in this project, as "a clustered-dot halftoning mimics the photoengraving process used in printing, where tiny pixels collectively comprise dots of various sizes" (Ulichney,

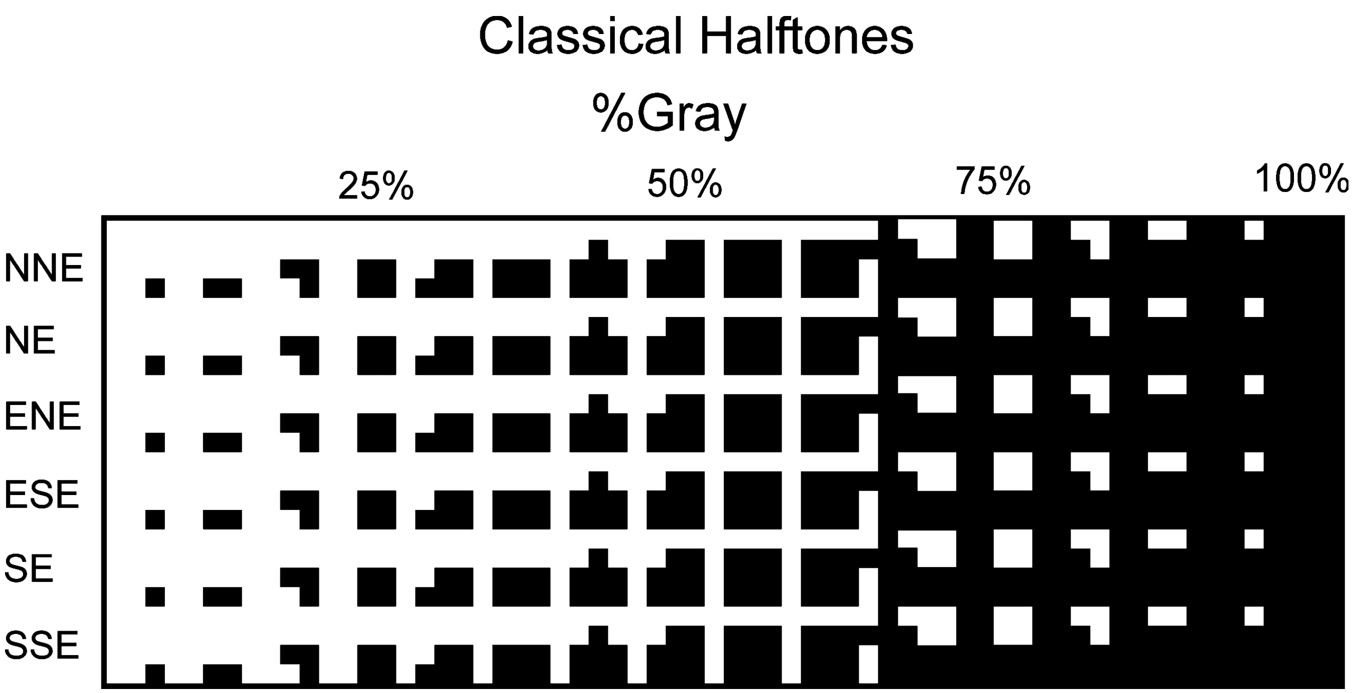


Figure 7. A classical clustered-dot ordered dither representing 16 shades of gray. (Modified from Ulichney (1987); Figure 5.5 on page 88).

1987, 3). This type of halftoning is referred to as the "classical", graphic arts, or printer's screen (see **Figure 7**).

This classical monochrome screen is often oriented at a 45° angle from the edge of the video screen or paper. This orientation is generally considered to result in the least visual disturbance or a minimum in perceptual sensitivity (Robinson *et al*., 1995; Ulichney, 1987). Sometimes a pattern is desirable. Computer graphic professionals uses clustered-dot line screens to create a halftone special effect (Ulichney, 1987). The linear dithering resulting in one pattern across the entire image, but adds no additional information to the display.

"Sometimes a pattern is desirable."

Computer graphic professionals have manipulated the orientation of individual halftones in various ways. Sloan and Wang (1992) oriented halftones based on the information contained in grayscale images. They

Figure 8. An image with halftones oriented in the direction of gradient of gray values (From Sloan and Wang (1992) with permission from SPIE).

choose locally different ordered dithers based on measures of local variations in values of gray (see **Figure 8**). Their rationale is that this method provides "an extra channel that we can use to convey information about the image" (149).

A more difficult problem is orienting halftones base on three dimensional (3D) geometries in virtual scenes. Sloan and Wang's (1992) research begins with a grayscale image, devoid of information on the 3D geometry captured in the image. Saito and Takahashi (1990) began with a 3D computer graphics model and stored orientation information in a series of geometric buffers (g-buffers). Information on many different geometric components of a virtual scene is often stored in this manner. This method was modified by Haeberli (1990) to orient the pattern of brushstrokes for virtual 3D scenes. Veryovka and Buchanan (1999) used g-buffers to orient halftones on an image of a 3D model based on its geometry. Haeberli (1990) and Veryovka and Buchanan (1999) used complex raytracing algorithms to define the geometry of the objects in the virtual scene.

"Haeberli and Veryovka and Buchanan used complex raytracing algorithms to define the geometry of the objects in the virtual scene."

The method outlined in this paper includes geometric information derived from a 2.5 dimensional surface, one in which each grid cell has a unique z-value. A hillshaded image can contain multiple grid cells of one shade of gray that represent any or all aspect directions. An example is a map of a conical hill under vertical illumination. The entire hill would be shaded with the same tone of gray. With a constant gray value, no pattern would be added to this hillshaded image with Sloan and Wang's (1992) method. The only geometric information needed to orient halftones to create patterns similar to hachure maps is the aspect of each grid cell. The methodology for creating such maps is described below.

"The only geometric information needed to orient halftones to create patterns similar to hachure maps is the aspect of each grid cell."

This project computes hillshading from DEMs and reclassifies the resulting 256 shades of gray into 16 equal interval classes. Aspect is calculated from the same DEMs and the resulting 360 degrees of aspect values are reclassified into 12 equal interval classes. The grids are converted into one point coverage containing hilllshading and aspect information. Each point then is displayed with a 4x4 bitmap based on its aspect and hillshading classes.

Bitmaps are constructed for each of the 192 combined classes. The number of black pixels (1-16) matches the classes of gray associated with hillshading. The patterns of the pixels are based on the aspect direction. North-south and east-west aspect patterns are not used, as the resulting linear patterns are too distracting. Identical patterns are used for aspect directions of northeast and southwest, etc. Thus only 96 unique bitmap patterns are constructed.

"The vertical and horizontal hachure textures cut across shades of gray."

Ordered clustering of individual halftones can approximate vertical or horizontal hachuring using the same bitmaps. The arrangements for various orientations to create a vertical hachuring effect are shown in **Figure 9** and those for a horizontal hachuring effect are shown in **Figure 10**. Both figures show only the 96 unique halftones used to pattern the eastern aspect directions; the western aspect directions would be a reflection of these across a vertical plane.

These halftone patterns are applied first to a gridded hemisphere composed of 170x170 grid cells and shaded from the northwest. A hemisphere is used because of its smooth variations in aspect and hillshading, as well as its easily recognized form. Any discontinuities in halftone shades or patterns should be obvious on the hemisphere. The hemisphere is dithered with the classical halftone pattern in **Figure 11**, the vertical halftone pattern in **Figure 12**, and the horizontal halftone pattern in **Figure 13** (compare with **Figures 7, 9 and 10** respectively). Although the 12 classes of aspect can be discerned, distracting apparent changes in shades of gray between various aspect classes are minimal.

The vertical and horizontal hachure textures cut across shades of gray. This result is expected, as halftone orientation is based on aspect direction and gray tone is based on hillshading. If Sloan and Wang's (1992) method are applied to this image of a hemisphere, halftone patterns would follow changes in gray and be coincident with the concentric areas defined by hillshading.

"Areas of moderate gray tone (i.e. moderate number of black halftones) show pattern most clearly."

These same halftone patterns are then applied to a DEM of a portion of the Sweet Grass Hills of north-central Montana illuminated from the northwest. This area was chosen because the landform shows frequent variations in aspect direction associated with peaks, ridges, and valleys. Also, the area is covered by a good quality DEM. The portion of this DEM is composed of 170x170 grid cells, each 30x30 meters. **Figure 14** shows the original hillshading of the DEM with all 256 shades of gray. **Figure 15** shows the shades of gray reclassified into the 16 classes used in the dithering processes. Although only 16 shades of gray are displayed, each is dithered at a much finer resolution by the printing process than the dithered images in subsequent figures.

The DEM is dithered with the classical halftone pattern in **Figure 16**, the vertical halftone pattern in **Figure 17**, and the horizontal halftone pattern in **Figure 18** (again, compare with **Figures 7, 9 and 10** respectively). Although the dithered appearance is inescapable, all three figures match the 16 shades of gray in **Figure 15**.

If areas of similar aspect in **Figure 17** are large enough, the method effectively applies a vertical halftone pattern. Areas of moderate gray tone (i.e. moderate number of black halftones) show pattern most clearly. Regional and local peaks, as well as stream intersections, appear as divergent or convergent patterns. Some stream valleys and ridgelines appear as a

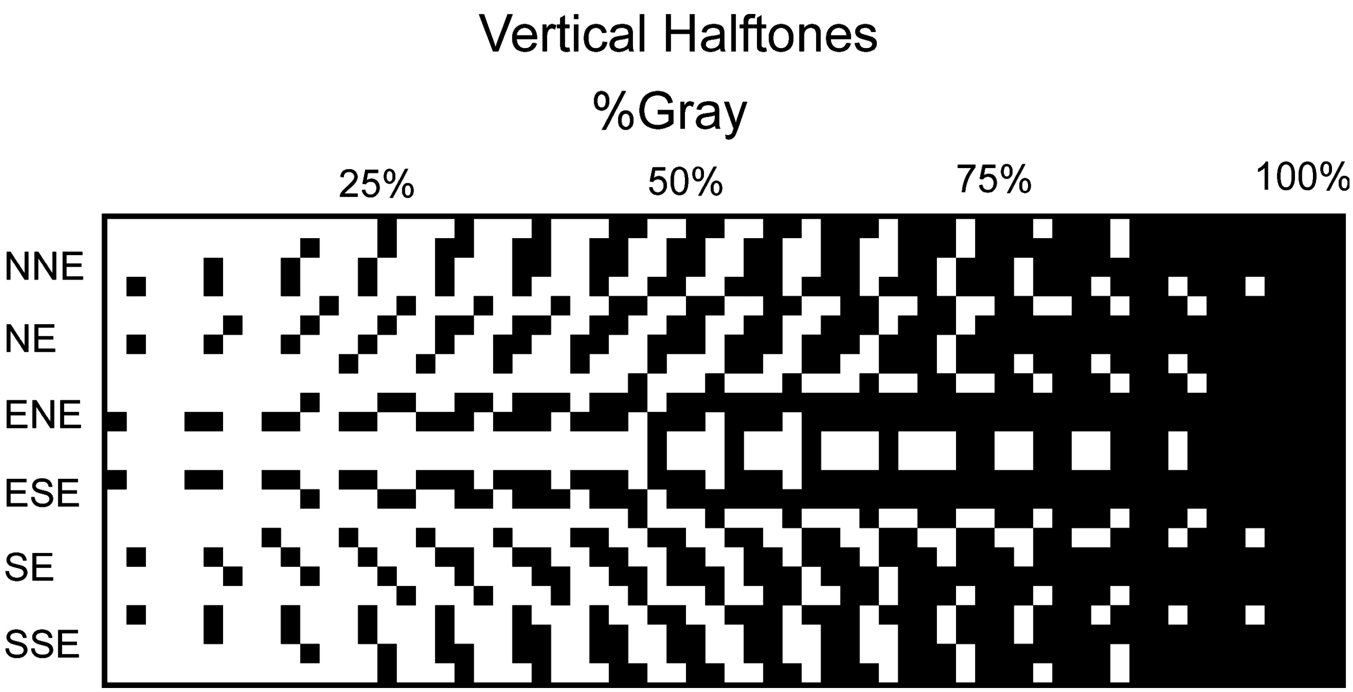


Figure 9. A dithering pattern for halftones oriented in the direction of aspect.

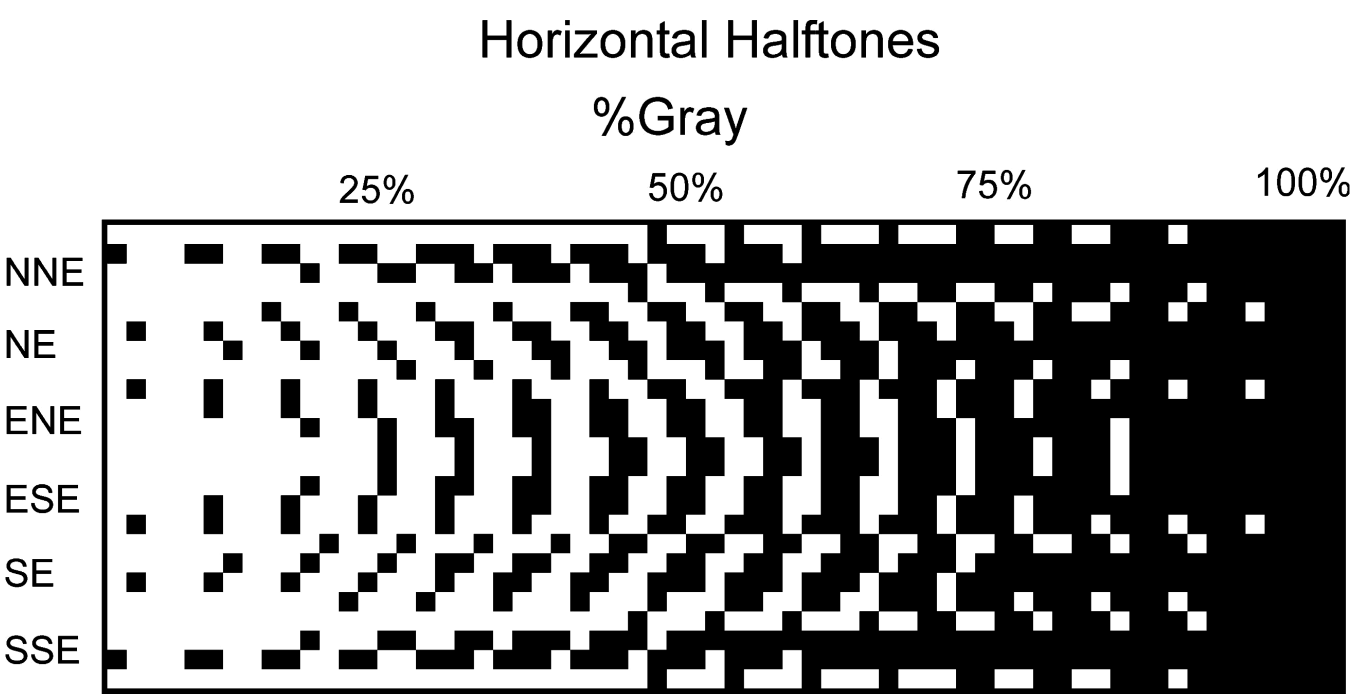


Figure 10. A dithering pattern for halftones oriented perpendicular to the direction of aspect.

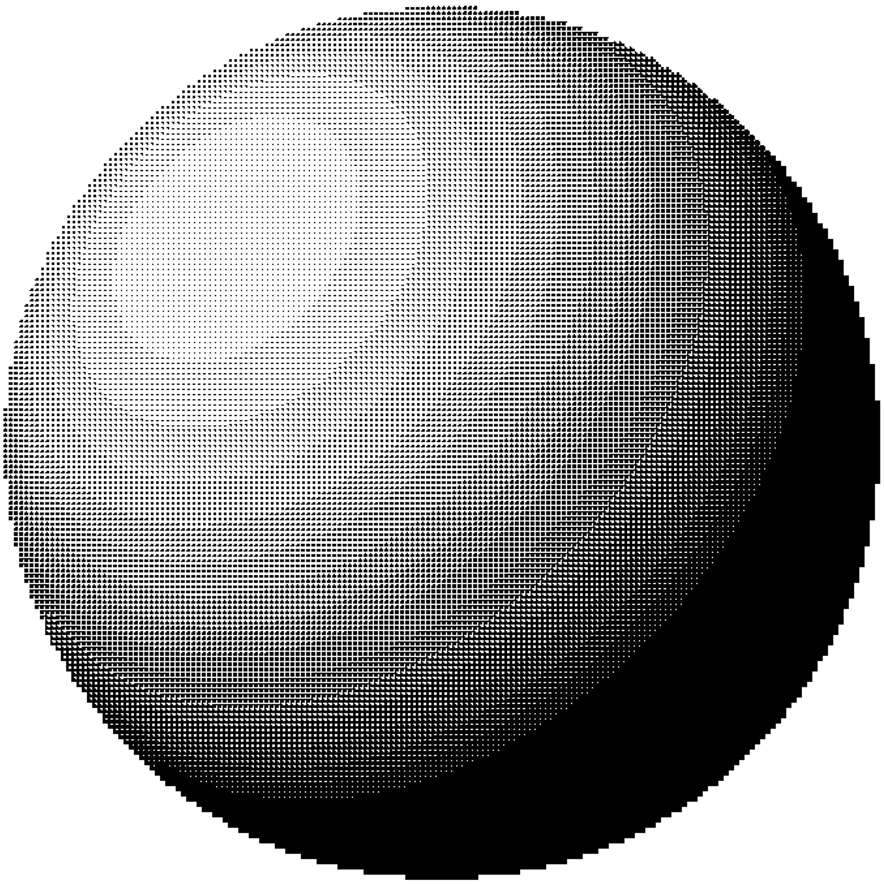


Figure 11. A hillshaded hemisphere dithered with the classical halftone pattern from ***Figure 7****.*

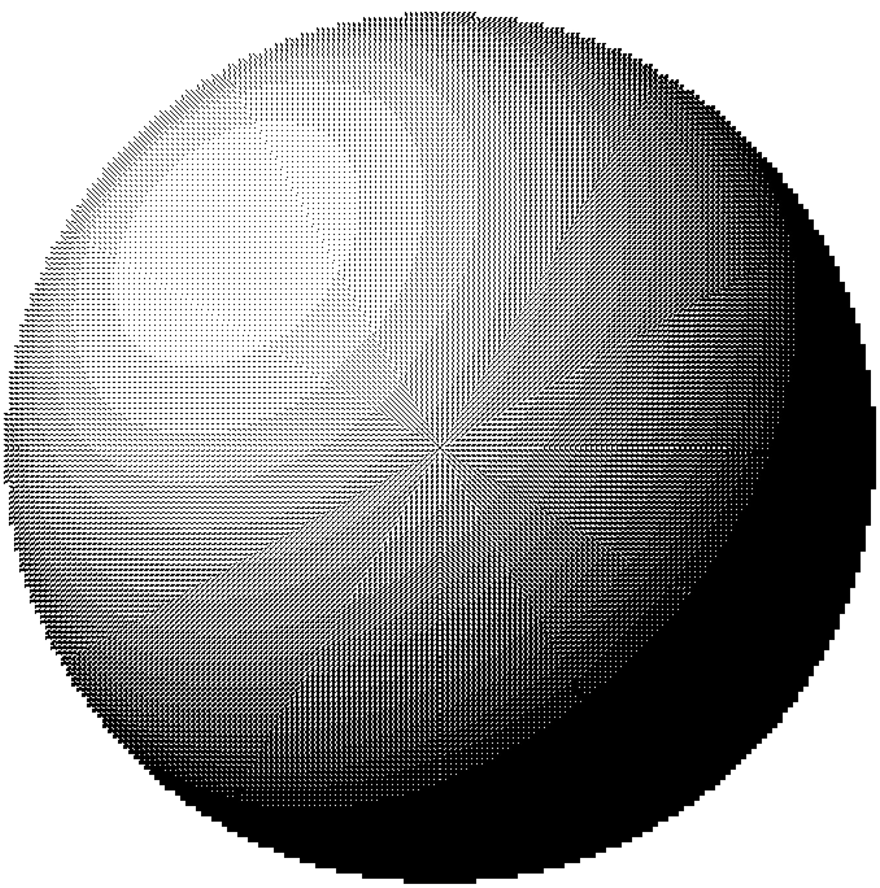
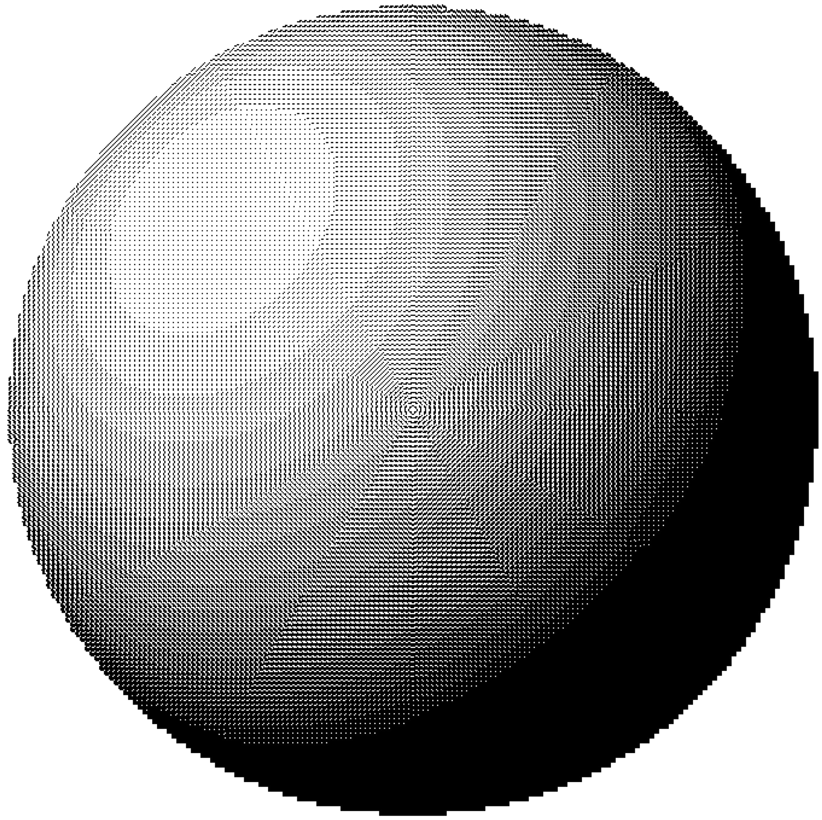


Figure 12. A hillshaded hemisphere dithered with the halftone pattern from ***Figure 9*** *oriented in the aspect direction.*

Figure 13. A hillshaded hemisphere dithered with the halftone pattern from ***Figure 10*** *oriented perpendicular to the aspect direction.*



single line, with divergent halftone patterns on either side. Valleys tend to form V's pointing downstream, as is the case with traditional hachures.

Figure 18 shows patterns associated with horizontal hachures. Areas of similar aspect with moderate hillshading values again show the most obvious patterns. Regional and local peaks appear as nearly closed forms. Valleys are delineated by a series of V's that point in the upstream direction, as is the case with horizontal hachures and contours.

Strongly and weakly illuminated landforms are not well delineated by this method. For example, strongly illuminated valleys are light gray and are composed of few black halftones. These few black squares are unable to create the patterns described above. In a similar manner, nearly black area with a few white squares would not be effective at creating pattern.

"Error diffusion would improve the look of such images, but also introduces additional noise into the display."

DISCUSSION

Limiting the classes of aspect to 12 has a noticeable effect on the resulting maps (see **Figures 17 and 18)**. Overall, patterns tend to be angular rather than smooth. An obvious artifact results from aspect patterns of northeast, southwest, southeast, and northwest occurring exclusively over a large enough area. Examples of this artifact can be seen in the southeast and east-central portion of these maps. Such artifacts can be smoothed using various computer graphic techniques, the most common of which is error diffusion (Strothotte and Schlechtweg, 2000; Ulichney, 1987). Error diffusion would improve the look of such images, but also introduces additional noise into the display.

One way to improve the appearance of these maps would be to increase the resolution of the bitmaps that represent each point. If bitmaps were 16x16 pixels, 60 directions of aspect could be used (again avoiding

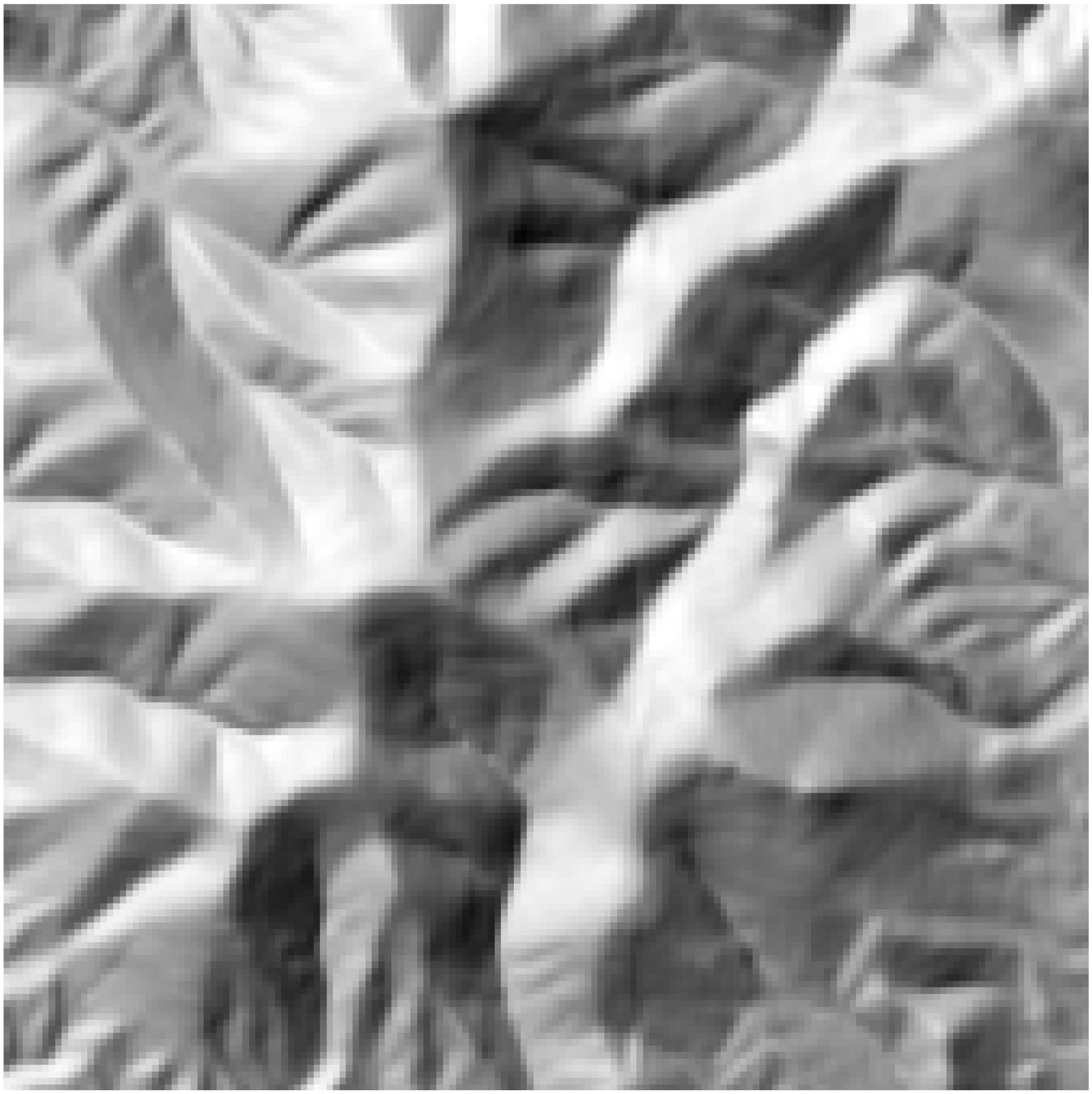


Figure 14. A hillshaded map of the Sweet Grass Hills of north-central Montana using 256 shades of gray.

north, south, east and west orientations). This would decrease the interval of each aspect class to 6° (from the interval of 30° in **Figures 17 and 18**). This method also would allow unique representation of 256 shades of gray, but require 3,840 unique bitmaps.

"An alternative approach would be to design and use a series of vector-based point symbols, with each point symbol precisely representing grayness."

An alternative approach would be to design and use a series of vector-based point symbols, with each point symbol precisely representing grayness. Such point symbols can be designed as characters in a true type font and converted into point symbols. Vector-based point symbols would be high resolution, unlike the pixellated bitmaps used in this project. Additionally, only 16 point symbols would be required to match shades of gray used in this paper, because all vector-based point symbols could be rotated by 1 degree increments of aspect. The effect would be similar to **Figure 4**, but grayness would more closely match exact hillshading values.

The issue with such an approach is that point symbols rotated around an axis at the center of a square grid cell can overlap neighboring point symbols. This overlap would decrease the overall darkness of the shad-

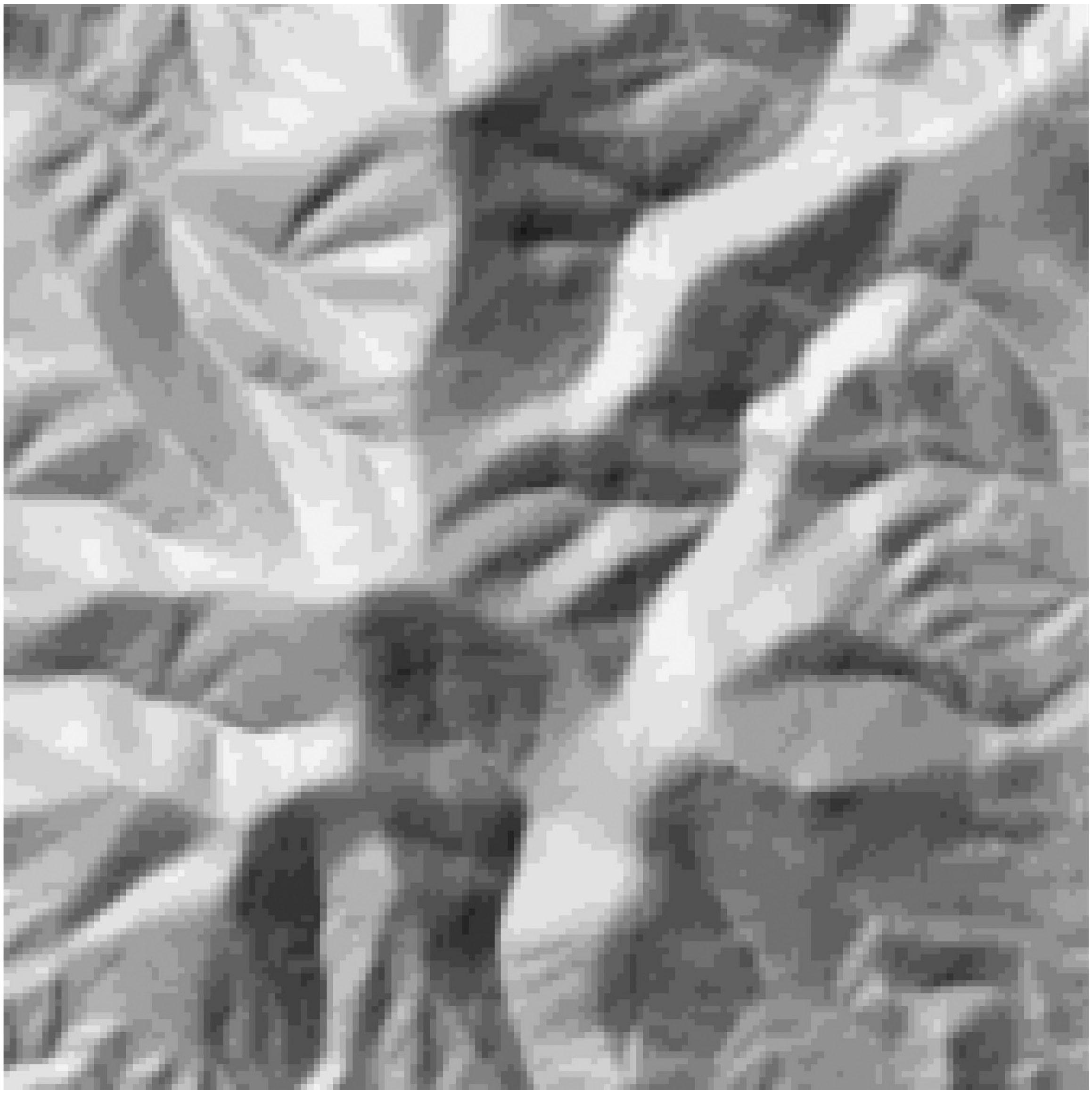


Figure 15. A hillshaded map of the Sweet Grass Hills of north-central Montana using 16 shades of gray.

ing over various parts of the map. A simple example showing one point symbol in two rows of square grid cells representing two aspect directions is illustrated in **Figure 19**. The relative brightness is increased by more than 20% in the lower row due to overlap of the black point symbol upon rotation.

The resolution of **Figures 17 and 18** is approximately 160 dots per inch (dpi) for individual black and white pixels. If each clustered dither is considered a dot, the resolution is approximately 40 dpi. The desired resolution is somewhat higher than 40 dpi, and was chosen to allow map users to resolve the shapes associated with the clustered pixels so that (s)he can identify the pattern.

"Cartographers have not studied the ability of map users to identify patterns of dithering in hillshaded maps."

Cartographers have not studied the ability of map users to identify patterns of dithering in hillshaded maps. Cartographers, however, have looked at the ability of map users to perceive pattern in gray area symbols composed of black dots on a white background. Although they were looking at a different issue, their results on the scale at which pattern can and cannot be perceived may serve as a starting point for the scale of display-

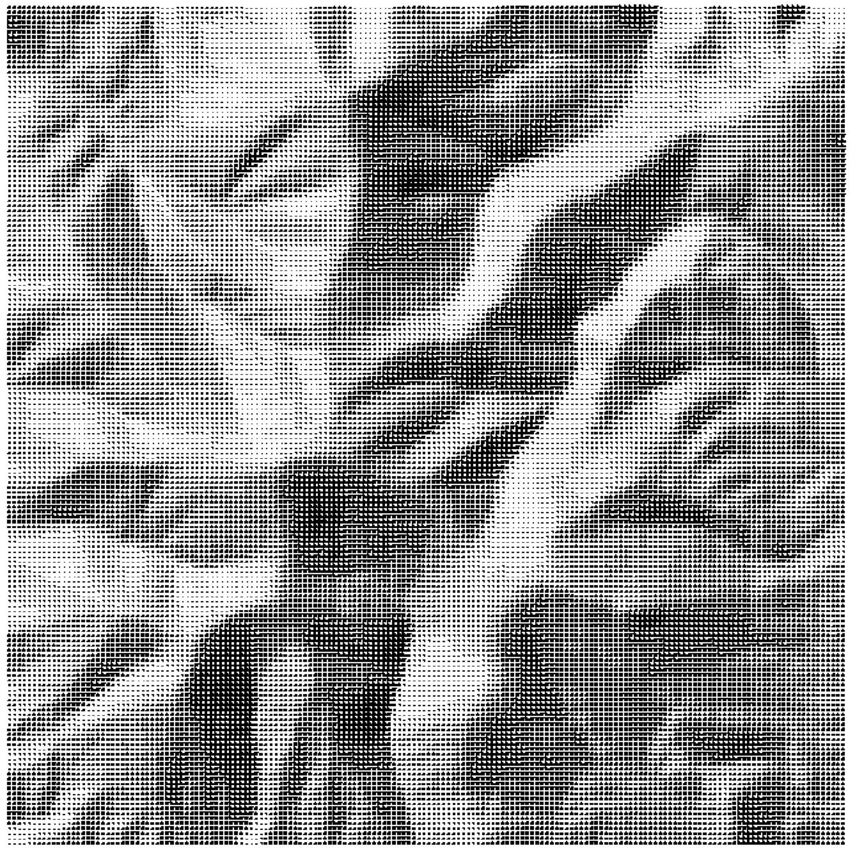


*Figure 16. The same hillshaded map in **Figure 15** dithered with the classical halftone pattern from **Figure 7**.*

ing oriented halftones with this method.

Castner and Robinson (1969) studied the map user's ability to discriminate a pattern in gray area symbols created from black dots on a white background. They determined that below 40 lines per inch (lpi = dpi) most map users perceived the patterns of dots and not the grayscales, above 75 lpi they perceived gray tones and not patterns, and between these resolutions they perceived both.

"MacEachren . . . distinguishes between the issues of discrimination, the ability to recognize a difference, and detection, the problem of discriminating between some signal and the background on which that signal appears."

MacEachren (1995) also discusses gray area tones comprised of dot patterns. He distinguishes between the issues of discrimination, the ability to recognize a difference, and detection, the problem of discriminating between some signal and the background on which that signal appears (p. 124). He identifies ambiguous patterns at resolution between 40 and 85 dpi, where fills can be seen as either shades of gray or textures (p. 125).

Figures 17 and 18 are meant to have such ambiguous patterns. The goal of this method is to create hillshaded maps in which the map user can see both shades of gray and patterns in the halftones that create the shading. An additional issue with ambiguous patterns with important implications to this method may be that such patterns can be seen as gray or textured, but not both at once (MacEachren, 1995, 125).

CONCLUSIONS

This paper explains a method for hillshading that uses different patterns for 12 classes of aspect, each comprised of 16 shades of gray. The clustered-dot ordered dithers form patterns similar to those in vertical and horizontal hachure maps. Dithering with this method allows patterns

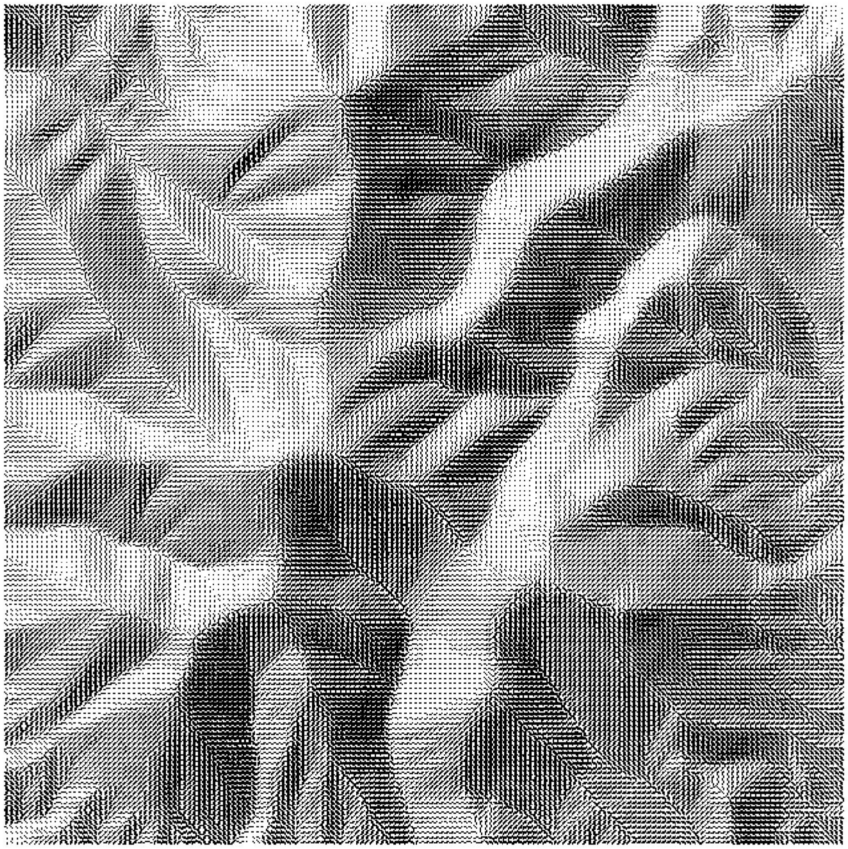


Figure 17. The same hillshaded map in ***Figure 15*** *dithered with the halftone patterns from* ***Figure 9*** *oriented in the aspect direction.*

at the resolution of the input data, while matching classes of gray determined by hillshading.

Creating accurate hillshaded maps with continuous shades of gray has twice been a goal of cartographers, first with hand-rendered and then with computer-generated maps. The computer reproduction technique of halftoning adds a new control to shading. Halftones, the elemental building block of computer hardcopy displays, can be used to create a pattern, while maintaining the grayscales determined from hillshading.

ONLINE RESOURCES

The author welcomes users to experiment with this method and the resulting images. Copies of the bitmaps used for **Figures 7, 9 and 10** and the ESRI ArcView legend files (.avl extension) used to create **Figures 16, 17, and 18** can be downloaded from http://www.mbmg.mtech.edu/gis_hillshading.htm. This website also includes GIS based methods for automating the methods of hachure mapping and those developed by Tanaka (1932 and 1950) that are discussed in this paper. For other recent applications of hillshading methods, see the NACIS Shaded Relief Homepage maintained by Tom Patterson at http://www.nacis.org/cp/cp28/resources.html.

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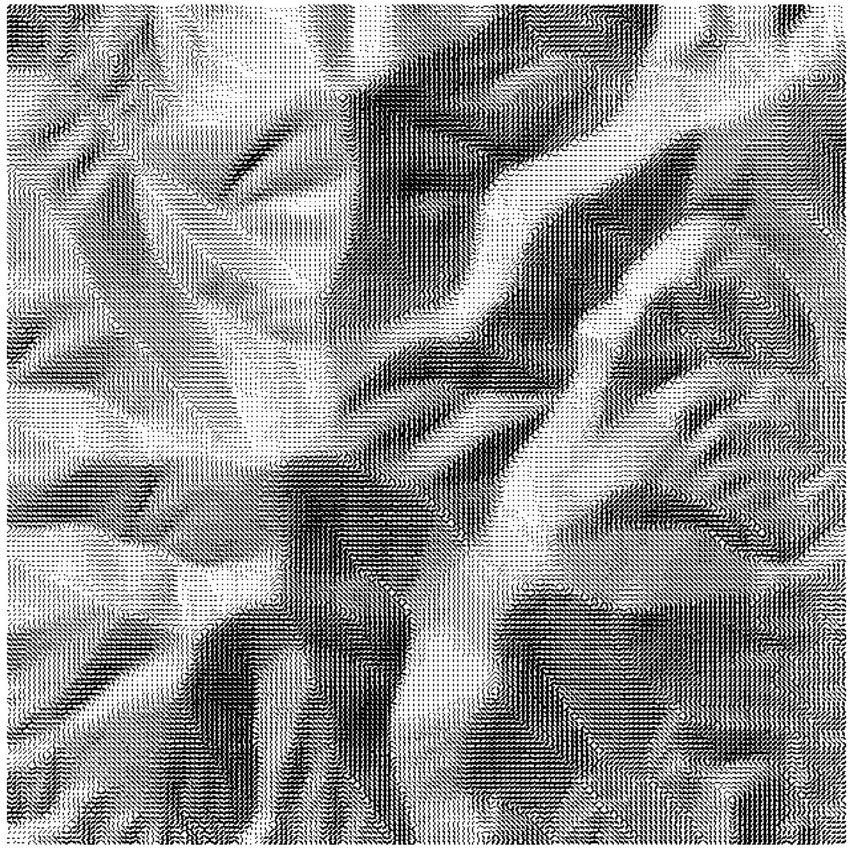

*Figure 18. The same hillshaded map in **Figure 15** dithered with the halftone patterns from **Figure 10** oriented perpendicular to the aspect direction.*

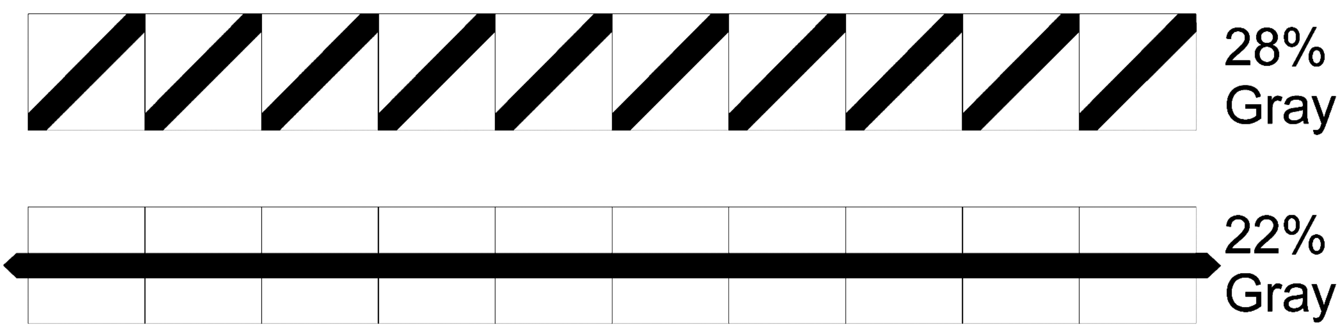

Figure 19. Two rows of vector-based point symbols centered in square grid cells. Symbols in the second row have been rotated 45° clockwise around the center of the grid cells. Overlap of the black point symbols in the second row results in a decrease in overall darkness between rows from 28% gray to 22% gray, a relative change of more than 20%.

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