Visualizing Method-Produced Uncertainty in Isometric Mapping

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Isometric mapping, while highly uncertain, continues to be a preferred mapping method for continuous data in many of the physical and social sciences. Isometric method-produced uncertainty refers to the various map representations that result when different methods and/or specifications are used in the mapping process. This paper examines ways to communicate the nature and magnitude of isometric method-produced uncertainty to map readers so that they are encouraged to be uncertain when it is warranted. As a case study, we consider an extensive set of plant hardiness zone maps that result when different interpolation methods and sampling resolutions operate on the same set of data. Our results show that slightly different choices in the mapping process can result in very different looking isometric maps, and suggest that the manifestations of method-produced uncertainty are not as systematic, or straightforward, as suggested by interpolation accuracy assessments. We then explore the use of two existing visualization techniques, flickering and transparency, to communicate the nature and magnitude of isometric method-produced uncertainty.

Key Words: Map uncertainty, isometric mapping, map animation, visualization

INTRODUCTION

The fact that one can never be certain about the precision or accuracy of maps, nor their underlying data, is inextricably bound to cartography. As is the case with any other communication medium, mapping is afflicted with misconceptions, misinterpretations, mistakes, and method-produced error. A great deal has been written and published about cartographic uncertainty, at times using synonyms such as accuracy, quality, error, and reliability (e.g., Buttenfield, 1993; Hunsaker et al., 2001; Hunter and Goodchild, 1996; MacEachren et al., 1998). Quite often, uncertainty is posed as not just an inherent product of map making, but as a 'quality' which has negative impact, and as an explanation for some of the frailties of maps. In that sense, uncertainty is an unavoidable byproduct of mapping geographic reality at scales that demand reduction in certainty, among other things. None of this is news to cartographers, but it may be a revelation to people who use our maps.

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Cartographic uncertainty exists as one of the costs we incur in map visualization, but map users are rarely encouraged to feel uncertain about the maps they view. We often lecture students in our classes about map fallibilities, and we may write about numerical expressions of error or reliability. That said, it would be understandable if cartographers expected the public to have an inbuilt wariness of maps. However, this is probably not the case because map readers are not usually informed in an explicit way that maps have shortcomings that can't be entirely remedied. In the case of this research, we maintain that people who read maps are not normally instructed about the meaning of uncertainty, or how to under-

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stand uncertainty. The goal of this paper is to examine some possible ways to inform map readers visually about uncertainty using static as well as animated techniques.

Method-Produced Uncertainty in Isometric Mapping

Isometric mapping has a strong tradition and extensive history in the social and physical sciences, and is a preferred method for mapping distributions of continuous phenomenon measured with interval or ratio-scaled data (see Robinson, 1961). In isometric mapping, real data values, or control points, are used to develop a three dimensional surface that is visualized by two dimensional quantitative line symbols. While once a manual process, the majority of contemporary isometric maps are created using computer-based methods (Dent, 1999; see also Mulugeta, 1996). In automated interpolation, a continuous grid of data values is derived from a non-continuous distribution of control points. Following interpolation, isolines are placed according to specified intervals.

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While all maps have uncertainty, the uncertainty associated with isometric mapping is exacerbated by the fact that the majority of data values shown on a map are estimated from a limited number of control points. Furthermore, isometric maps tend to be very unstable, with different isoline placement when different techniques or specifications are applied to the same set of control points (see MacEachren and Ganter, 1990). To date, most treatments of isometric uncertainty have focused on map error. For example, Morrison (1971) defined three sources of method-produced error in isarithmic mapping: (1) the number of control points used, (2) the distribution of those control points, and (3) the interpolation method. Similarly, Robinson et al. (1995) defined several additional sources of error relating to data quality, class interval assignment, and the implied accuracy of the mapping concept itself.

By isometric method-produced uncertainty, we mean the various map representations that can result when different methods or specifications are used to map a given set of data. We use the term uncertainty, rather than error, because the majority of locations on a statistically derived surface cannot be validated. Consequently, the amount of error on a particular map will never truly be known. We suggest that method-produced uncertainty is manifested in three different but related ways: (1) interpolation accuracy, (2) visual stability, and (3) information stability, as discussed below.

"We use the term interpolation accuracy in reference to the extent that an interpolated surface deviates from the original set of control points."

Interpolation Accuracy

We use the term interpolation accuracy in reference to the extent that an interpolated surface deviates from the original set of control points. The majority of research concerned with interpolation accuracy has focused on assessing different interpolation methods in light of statistical accuracy. These studies have used several techniques, such as cross-validation (Isaaks and Srivastava, 1989), true validation (Voltz and Webster, 1990), and a variety of summary statistics to evaluate the accuracy of interpolated surfaces. Some early studies include Morrison's (1974) assessment of various interpolation methods and Dubrule's (1984) comparison of splines versus kriging for estimating well depth. Other studies conducted by soil scientists were concerned with the accuracy of different interpolation methods for predicting soil characteristics, such as moisture capacity (Van Kuilenburg et al., 1982), pH (Laslett et al., 1987), and clay content (Voltz and Webster, 1990). More recently, Declerq (1996) evaluated the accuracy of several interpolation methods using control point distributions with very different characteristics. By and large, these studies have shown that kriging and inverse distance weighting (IDW) tend to minimize interpolation error, but that results will vary depending on the nature and distribution of control point data.

Visual Stability

Visual stability is related to the concept of map stability (Muehrcke, 1990). It refers to the extent that visually detectable differences are found between isometric maps when different interpolation techniques or specifications are applied to the same set of control points. Visual stability is based on the premise that greater variability is associated with greater uncertainty as to which map best portrays a particular geographic phenomenon. Visual stability is scale dependent because differences that are visible at a large scale may be virtually indistinguishable as scale becomes smaller. For example, MacEachren and Ganter (1990) compared the visual stability of isoline and three-dimensional fishnet patterning when different sampling resolutions are applied to the same set of data. They found that patterning with isoline representations tends to be much less stable than with fishnet representations, and suggested that climatologists might benefit from using an alternative visualization method for representing their data.

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Information Stability

By information stability, we mean the extent that the information shown on an isometric map changes when different interpolation techniques or specifications are applied to the same set of data. Information stability is similar to visual stability, but is not scale dependent if information is extracted from the map using non-visual techniques. Information stability is especially important when isometric maps are used in a Geographic Information System (GIS) for vector overlay, for coding point locations, or when they are viewed using an Internet Mapping Server (IMS), where users may have the ability to view maps at inappropriately large scales. For example, if point is located over a data island that is not visible at an intended scale, the point will be coded according to that data island, despite its size or visibility.

Communicating Isometric Uncertainty

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Despite the ambiguities discussed above, isometric mapping continues to be a preferred mapping method for many physical sciences, particularly in the fields of climatology and meteorology (see Dibiase et al., 1994). Map makers seldom, if ever, convey the nature and magnitude of method-produced uncertainty to map users, and this is especially problematic when isometric maps are used as a primary analytic tool (see MacEachren and Ganter, 1990).

Summary statistics, such as Root Mean Square Error (RMSE), are typically used to communicate interpolation accuracy (USGS, 1997). However, a number of problems arise when a single summary statistic is used to communicate the uncertainty associated with a particular map. First, summary statistics do not indicate how uncertainty is distributed from place-to-place. For example, Shortridge (2001) noted that a single portion of a digital elevation model could account for the majority of error reflected in a summary statistic. Second, two maps generated from the same set of control point data can have similar accuracy values, but show very different patterning (Declerq, 1996). Even when control points are removed from the sample for subsequent validation, interpolation accuracy remains a statistically informed guess. Finally, summary statistics generally estimate the accuracy of an interpolated grid and do not necessarily account for uncertainty in isoline placement. Map users rely on the position of isolines to extract information, not on the gridded surface that has been evaluated.

Very few alternatives to summary statistics have been proposed to express method-produced uncertainty in isometric mapping. A notable exception includes Robinson et al.'s (1995) suggestion of using isoline smoothing to promote wariness about the accuracy of line placement. They discussed this technique in the context of conceptual error, or "the validity of the concept presented by the map" (Robinson et al. 1995:514-515). As an example, they considered an isometric map representing the distribution of mean temperature data and suggested that "Cartographers can overcome the effects of these errors and inconsistencies from one part of an isarithmic map to another by smoothing the isarithms" (Robinson et al. 1995:515). Also related, but not specific to isometric mapping, is van der Wel's (1993) use of sliders to visualize uncertainty thresholds for categorical boundaries via line width and blurring (see MacEachren 1995:443) and Hengl et al.'s (2004) visualization of interpolation uncertainty using different confidence thresholds.

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Given the reality of method-produced uncertainty in isometric mapping, it is important to understand this uncertainty, and to communicate its existence and extent to map users, especially if a map is used for analytic purposes, or to assist in policy decision-making. And therein lies the challenge. Exactly how are we to present the notion and magnitude of isometric uncertainty to map users? How can we facilitate the appropriate use of the isometric maps that we make? The purpose of this research is to examine the use of existing visualization techniques for communicating isometric uncertainty in order to inform map readers about how to interpret a particular map. The techniques that we consider include (1) flickering and (2) transparency, as discussed below.

The technique of flickering, attributed to MacEachren (MacEachren et al., 1993), draws from the concept of alternating syntagms, where different attributes of the same place are alternately displayed in register (see Monmonier 1992). Flickering, as applied by MacEachren (1995), involves the use of non-temporal animation to alternate between two or more maps so that a map reader can consider multiple pieces of information simultaneously. As an example of how flickering can be used to communicate uncertainty, MacEachren (1995) considered dissolved nitrogen surfaces for the Chesapeake Bay over a six-year period. He suggested that for each month, maps can be generated using different interpolation methods and "when these flickering images are run in sequence as an animation, they should provide both a reliability assessment of maps at each time period, and a way to assess changes in pattern stability over time" (MacEachren 1995, 447). We use the technique of flickering in a similar way, that is, to assess and visualize uncertainty in isometric boundaries when different interpolation techniques are applied to the same set of data.

The technique of using transparency to communicate uncertainty draws from MacEachren's visual variable "focus" (MacEachren 1992), which he later described in terms of "clarity" (MacEachren 1995). MacEachren subdivided clarity into three visual variables, including crispness, resolution, and transparency. The term transparency, when used to visualize uncertainty, refers to a "fog" that differentially obscures the map theme based on data uncertainty (see MacEachren 1992:15). Rather than using a transparent fog to mask uncertain portions of a map, we use transparency as a technique to simultaneously display alternate isometric maps that result when different interpolation techniques and parameters are applied to the same set of data. In what follows, we apply these two techniques to see if they are useful for exploring and visualizing method-produced uncertainty in isometric mapping. First, however, it is necessary to investigate the various manifestations of isometric uncertainty for a particular set of data.

As a case study, we consider an extensive set of plant hardiness zone maps that result when different interpolation methods and sampling resolutions operate on the same set of control points. The most fundamental choices that affect the outcome of an isometric map are the selection of an interpolation method and gridding interval. Gridding interval refers to the distance separating nodes on an equally spaced grid used for interpolation. Kriging and IDW are considered here because they have been shown to perform better in terms of interpolation accuracy and are the most common methods used (see Lam, 1983).

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Plant Hardiness Zones

Plant Hardiness zone maps, intended to assist the public in planting appropriate vegetation, are found in a variety of textbooks, growing manuals, and classrooms throughout North America. Currently, the most widely used plant hardiness zone map was issued in 1990 by the USDA (Cathey 1990), and uses isolines to divide hardiness zones according to average annual low temperatures. This map is similar to previous versions (e.g., USDA, 1960) that follow hardiness zone map conventions. These conventions include (1) the use of average low temperatures recorded by weather stations to define hardiness zones, (2) the use of ten plant hardiness zones based on isolines placed at 10˚ F intervals, and (3) the use of a spectral color scheme to distinguish between ordinal hardiness zone values. We recognize that the use of alternative methods, such as incorporating elevation data, might improve the accuracy of hardiness zone maps (see Veve, 1994). However, our intent is not to develop better methods for making hardiness zone maps, but rather, to investigate ways to explore and communicate method-produced uncertainty in hardiness zone maps that have been created using conventional means.

For this study, we consider average annual low temperature readings archived by the National Climatic Data Center for 4,799 weather stations located within the conterminous United States (Figure 1). The average annual low temperature readings are based on temperature extremes from 1990 to 2000, with the number of observations ranging from 2 to 11 years for individual weather stations. The positional accuracy of control points is limited to 0.01 degree latitude and longitude. The distance between control points, as determined by Delaney triangles, varies from 1.8 to 385.3 km, with spacing between control points generally being greater in the mountainous west. The mean distance between control points is approximately 46 km.

". . . a total of 354 plant hardiness zone maps were created . . ."

With these data, a total of 354 plant hardiness zone maps were created in Surfer 7 (Golden Software 1999): 177 maps using kriging, and 177 maps using IDW. For each interpolation method, all default options were accepted, with the exception of the gridding interval. The default option in Surfer 7 uses ordinary, point kriging, and considers all of the control point data for interpolating each grid node. The default option relies on a linear variogram model taking the following form (Golden Software, 1999):

$$\gamma(h) = C_0 + S \cdot h$$

where

$\gamma(h)$	is the semivariance
C_0	is the unknown nugget effect
S	is the unknown slope
h	is the lag distance

To solve for (h), C_0 and S are determined as follows:

$$S = \max\left[\frac{Var - G_{nn}}{D_{ave} - D_{nn}}, 0\right]$$

$$C_0 = \max\left[\frac{G_{nn} \cdot D_{ave} - Var \cdot D_{nn}}{D_{ave} - D_{nn}}, 0\right]$$

where:

D_{nn}	is the average distance to the nearest neighbor
D_{ave}	is the average inter-sample separation distance
G_{nn}	is one half the averaged squared difference between nearest neighbors
Var	is the sample variance

The IDW option in Surfer 7 defaults to a weighting power of 2, with no smoothing. Like kriging, all control points in the default option are considered for interpolating each grid node. The equation for IDW is as follows (Golden Software 1999):

$$Z_{new} = \frac{\sum\left(Z_i / d_i^{\,wt}\right)}{\sum\left(1 / d_i^{\,wt}\right)}$$

where:

Z_i	is the control point z value
Z_{new}	is the interpolated z value
d	is the distance
wt	is the weighting power

For each method, 177 interpolated grids were created, each employing a different gridding interval ranging from 100 km to 1 km (Table 1). From 100 km to 23.3 km, all possible intervals allowed by Surfer were used. A sample of grid sizes were selected at 1-km increments for gridding intervals smaller than 23 km, because using every possible interval after that point would have required an overwhelming amount of production time with very little gain in information. For each interpolated grid, an isometric map was made in Surfer following plant hardiness zone map conventions.

Exploring Method-Produced Uncertainty in Plant Hardiness Zone Maps

As discussed at the outset, method-produced uncertainty in isometric maps is manifested in at least three different but related ways, including

Figure 1. Location of 4,799 weather stations considered for plant hardiness zones.

interpolation accuracy, visual stability, and information stability. Here, we explore method-produced uncertainty in plant hardiness zones given different choices in interpolation method and gridding interval.

Interpolation Accuracy

We evaluated the accuracy of each interpolated grid by computing RMSE values based on the difference between predicted and known temperatures for each of the 4,799 control points. RMSE values report the standard deviation of residuals (the difference between known and predicted values), and provide an estimate of how well an interpolated grid corresponds to the data used to create it. While a variety of summary statistics have been used to evaluate interpolation accuracy, we chose RMSE because it is relatively easy to compute, and significantly easier to understand than other methods.

"We evaluated the accuracy of each interpolated grid by computing RMSE values based on the difference between predicted and known temperatures for each of the 4,799 control points."

RMSE is derived using the following equation:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}\left(y_i - y_j\right)}{N-1}}$$

where:

y_i is the predicted temperature value

y_j is the known temperature value

N is the number of sample points

Table 1 reports the RMSE values generated for each interpolated grid. RMSE values decrease in a systematic, non-linear manner, with interpolation accuracy decreasing more rapidly as gridding intervals become finer (Figure 2). Figure 3 shows the distribution of residuals for selected gridding intervals, further emphasizing this trend.

There are several ambiguities in using this method to estimate interpolation accuracy. Because kriging and IDW, as applied here, operate as

Frame #	Gridding Interval (m)	IDW RMSE (ºF)	Krig RMSE (ºF)	Frame #	Gridding Interval (m)	IDW RMSE (ºF)	Krig RMSE (ºF)	Frame #	Gridding Interval (m)	IDW RMSE (ºF)	Krig RMSE (ºF)
1	100766	3.37	3.31	61	43782	2.56	2.25	121	27923	2.06	1.75
2	98622	3.38	3.30	62	43320	2.57	2.26	122	27755	2.04	1.73
3	96567	3.43	3.34	63	42918	2.57	2.22	123	27590	2.01	1.71
4	94596	3.32	3.23	64	42525	2.53	2.22	124	27427	1.99	1.70
5	92704	3.32	3.15	65	42138	2.52	2.22	125	27266	2.03	1.71
6	90887	3.36	3.22	66	41758	2.50	2.20	126	27106	1.99	1.68
7	89139	3.31	3.21	67	41386	2.54	2.23	127	26949	1.97	1.67
8	87457	3.32	3.26	68	41019	2.52	2.21	128	26793	1.95	1.65
9	85837	3.25	3.11	69	40660	2.44	2.14	129	26639	1.96	1.66
10	84277	3.25	3.14	70	40306	2.47	2.16	130	26487	1.96	1.66
11	82772	3.25	3.14	71	39959	2.48	2.18	131	26336	1.96	1.66
12	81320	3.19	3.07	72	39617	2.46	2.13	132	26187	1.95	1.65
13	79918	3.19	3.07	73	39281	2.44	2.12	133	26040	1.93	1.64
14	78563	3.17	3.02	74	38951	2.44	2.11	134	25895	1.91	1.62
15	77254	3.13	2.97	75	38627	2.40	2.10	135	25751	1.91	1.61
16	75987	3.11	2.99	76	38307	2.42	2.11	136	25609	1.93	1.63
17	74762	3.13	2.98	77	37993	2.39	2.11	137	25468	1.92	1.63
18	73576	3.12	2.96	78	37684	2.38	2.06	138	25329	1.94	1.65
19	72425	3.06	2.89	79	37381	2.38	2.07	139	25191	1.91	1.62
20	71311	3.02	2.84	80	37081	2.36	2.07	140	25055	1.89	1.59
21	70230	3.06	2.88	81	36787	2.34	2.04	141	24920	1.88	1.59
22	69182	3.08	2.92	82	36497	2.34	2.03	142	24787	1.87	1.59
23	68165	3.03	2.85	83	36212	2.34	2.02	143	24655	1.88	1.58
24	67177	3.02	2.83	84	35932	2.34	2.02	144	24525	1.91	1.61
25	66217	3.02	2.81	85	35655	2.32	2.01	145	24396	1.88	1.57
26	65285	2.99	2.80	86	35383	2.31	1.99	146	24268	1.84	1.56
27	64378	3.01	2.80	87	35115	2.30	1.98	147	24141	1.83	1.57
28	63496	2.94	2.72	88	34851	2.33	2.00	148	24016	1.83	1.55
29	62638	2.97	2.73	89	34591	2.34	2.00	149	23893	1.84	1.55
30	61803	2.96	2.74	90	34335	2.29	1.98	150	23770	1.82	1.54
31	60990	2.89	2.68	91	34082	2.28	1.96	151	23649	1.83	1.56
32	60197	2.89	2.65	92	33833	2.28	1.97	152	23525	1.82	1.54
33	59426	2.88	2.64	93	33588	2.26	1.94	153	23410	1.80	1.52
34	58673	2.86	2.62	94	33347	2.25	1.93	154	23292	1.78	1.50
35	57940	2.87	2.63	95	33108	2.24	1.93	155	22946	1.81	1.53
36	57225	2.84	2.59	96	32874	2.20	1.88	156	21967	1.73	1.45
37	56527	2.85	2.61	97	32642	2.21	1.89	157	20973	1.66	1.41
38	55846	2.85	2.60	98	32414	2.23	1.91	158	19979	1.61	1.37
39	55181	2.80	2.56	99	32189	2.22	1.89	159	18996	1.54	1.31
40	54532	2.81	2.56	100	31967	2.19	1.86	160	17966	1.46	1.24
41	53898	2.80	2.55	101	31784	2.18	1.87	161	16978	1.41	1.22
42	53278	2.81	2.56	102	31532	2.21	1.89	162	15983	1.33	1.16
43	52637	2.78	2.52	103	31319	2.18	1.86	163	15000	1.28	1.12
44	52081	2.75	2.51	104	31109	2.12	1.80	164	14003	1.18	1.06
45	51147	2.69	2.45	105	30901	2.15	1.82	165	12983	1.09	0.99
46	50936	2.69	2.43	106	30696	2.15	1.85	166	12008	1.04	0.95
47	50383	2.70	2.43	107	30495	2.15	1.83	167	11010	0.95	0.89
48	49841	2.73	2.47	108	30295	2.14	1.81	168	9989	0.86	0.83
49	49311	2.70	2.43	109	30098	2.08	1.78	169	9000	0.73	0.73
50	48792	2.65	2.35	110	29904	2.12	1.80	170	8005	0.64	0.67
51	48283	2.62	2.35	111	29713	2.13	1.82	171	7001	0.54	0.60
52	47786	2.67	2.41	112	29523	2.14	1.83	172	5996	0.46	0.54
53	47289	2.68	2.39	113	29336	2.11	1.79	173	5000	0.35	0.45
54	46820	2.66	2.35	114	29152	2.09	1.76	174	3999	0.26	0.37
55	47786	2.63	2.34	115	28970	2.08	1.78	175	3000	0.16	0.27
56	47289	2.63	2.33	116	28790	2.06	1.76	176	1999	0.09	0.19
57	45443	2.65	2.38	117	28612	2.00	1.71	-	1000	0.02	0.10
58	45002	2.62	2.32	118	28437	2.05	1.74	-	500	0.01	0.05
59	44569	2.57	2.27	119	28263	2.06	1.75				
60	44199	2.58	2.27	120	28092	2.04	1.73				

Table 1. RMSE values for kriging and IDW by gridding interval and animation frame number. The animation frame number refers to individual frames in Animations 1-3 (see Animations 1-3; Figure 2.)

exact interpolators, grid nodes occurring at the same location as control points will be assigned the same value as the control point (Lam, 1983). If a substantial number of grid nodes co-occur with control points, RMSE values will tend to underestimate interpolation error. While RMSE does not provide an exact indication of interpolation accuracy, fundamental

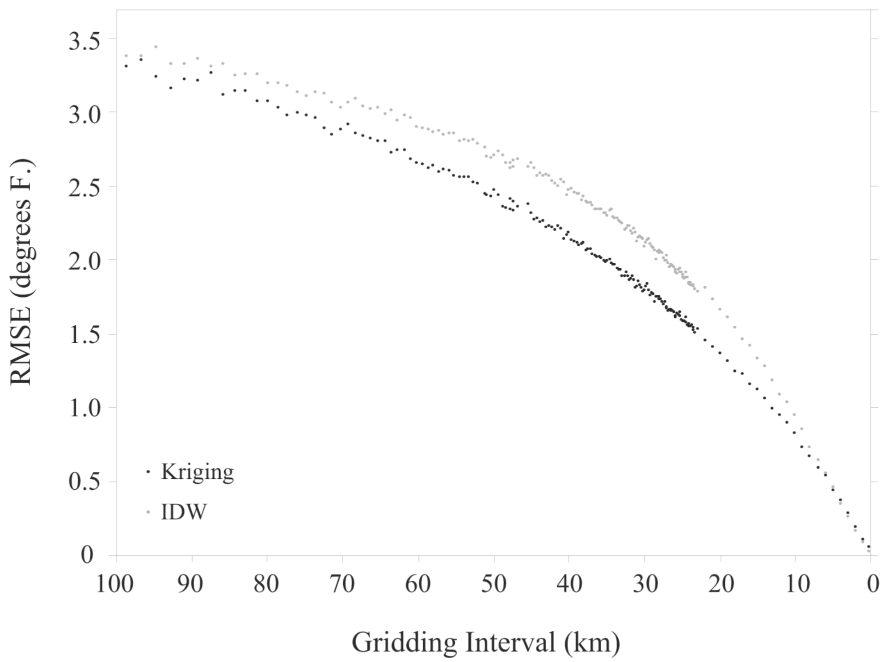

Figure 2. Scatter plot showing relationship between RMSE and gridding interval for kriging and IDW.

problems exist with all accuracy statistics that rely on the same set of data used for interpolation.

To see if the differences in interpolation accuracy between kriging and IDW are statistically significant, we performed a series of two-sample t-tests. The t-tests evaluate the null hypothesis that average error, or mean deviation, between the Kriging and IDW grid is similar. By average error, we mean the average difference between known values and predicted values on the interpolated surface. Figure 4 is a scatter plot diagram showing the t-test results by gridding interval and significance (p) value. The t-statistics report the strength of the difference in average error between Kriging and IDW. The graph shows that the strength of differences increases from 100 km to ~30 km, and then decreases dramatically until ~11 km. From 10 km to 1 km, the strength of difference rises dramatically. At the 0.05 significance level, these differences are statistically significant for gridding intervals ranging from 72 km to 14 km and gridding intervals ranging from 8 km to 1 km. The difference in average error is not statistically significant for gridding intervals ranging from 100 km to 73 km and for gridding intervals ranging from 13 km to 9 km. As can be seen in Figure 4, the majority of paired gridding intervals show statistically significant differences between Kriging and IDW interpolations.

"Although statistical significance provides a numerical measure of differences, it does not provide information about how map users may cognize visual patterns shown on isometric maps."

Although statistical significance provides a numerical measure of differences, it does not provide information about how map users may cognize visual patterns shown on isometric maps. Although it is beyond the scope of this research, it would be useful, for example, to empirically examine human responses to the visual effects of different gridding intervals. In this sense, subjects could compare frames for detectable differences they may see. The results of such an experiment could then be used in conjunction with the RMSE and t-test results to present both a statistical and cognitive view of isometric map patterns.

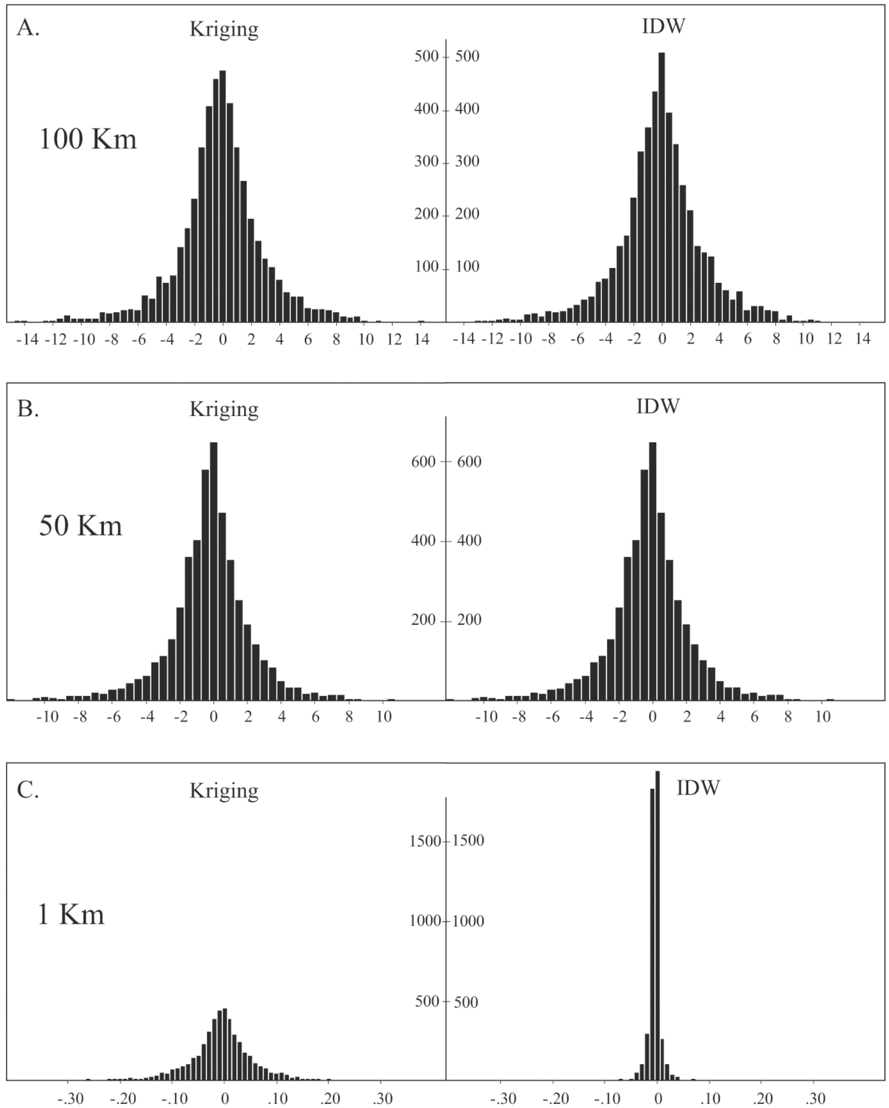

Figure 3. Distribution of the residuals (difference between predicted and observed values for each of the 4,799 weather stations) used for calculating RMSE for selected gridding intervals. a) Gridding interval is 100km. b) Gridding interval is 50 km. c) Gridding interval is 1 km.

Visual Stability

We display and evaluate visual stability in plant hardiness zones using the techniques of flickering and transparency. First, a non-temporal animation was created using Flash MX (Macromedia Corp. 2002) showing the hardiness zone boundaries for each interpolation method by gridding interval (Animations 1-3 [http://www.nacis.org/index.cfm?x=24]). The sequence is separated into three animations because of excessive file size. Compression was not used because we did not want to alter the original line geometry as determined by Surfer. The interface provides basic controls to play, stop, pause, and advance the animation sequence. We evaluated visual uncertainty by viewing the animation on a computer monitor at a scale of ~1:22,000,000, paying specific attention to line movement as gridding intervals decrease. Using this technique, visual uncertainty is not quantifiable, but we found it to be quite effective for (1) showing the variability in hardiness zone boundaries, and (2) for determining the gridding interval at which variability is no longer visible.

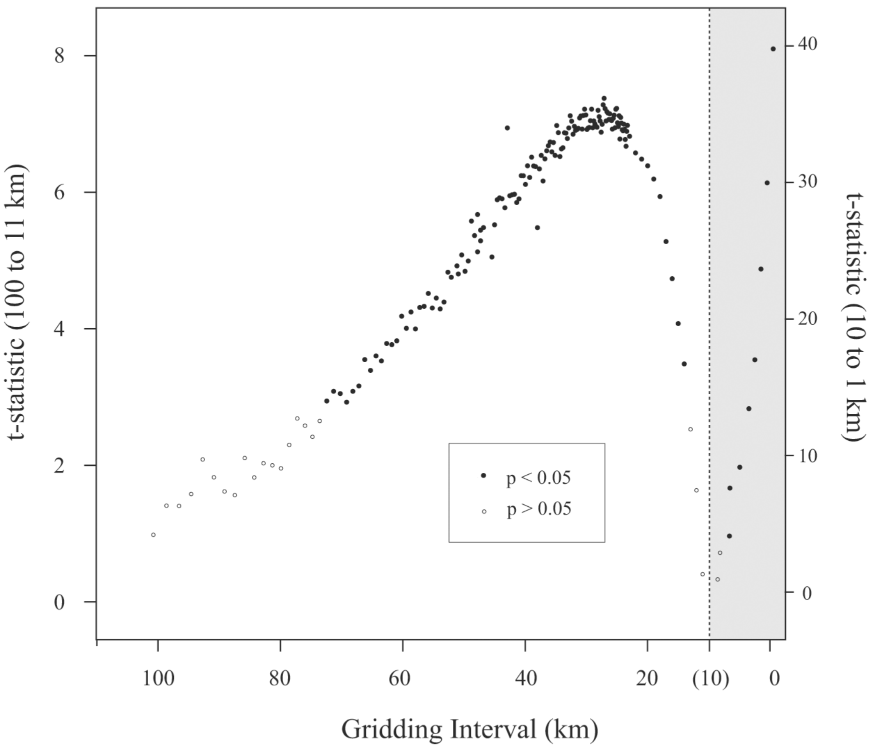

Figure 4. Scatter plot showing the result of 177 t-tests used to evaluate the null hypothesis that average error (difference between predicted and known values for 4,799 control points) is statistically similar for kriging and IDW when comparable gridding intervals are employed. The y-axis reports the t-statistic, or strength of difference between average error. The scale on the left indicates the t-statistic for gridding intervals ranging from 100 km to 11 km (white area), while the scale on the right indicates the t-statistic for gridding intervals ranging from 10 km to 1 km (gray area). Solid point symbols indicate a statistically significant difference, whereas circles indicate that there is not a statistically significant difference (● = 0.05).

The animations show that hardiness zone boundary variability is extreme for larger, and perhaps unrealistic, gridding intervals. For larger gridding intervals, boundaries shift dramatically and chaotically, even when similar gridding intervals are employed in the interpolation process. At a scale of ~1:22,000,000, stability for both interpolation methods occurs at a gridding interval of ~10 km. Variability in boundary placement does occur when finer intervals are employed, but this variability is difficult to see at this scale of observation.

The animation shows boundary variability between sequential gridding intervals, but it is not useful for assessing boundary differences that occur out of the animation sequence. Figures 5 and 6 show a variety of hardiness zone boundaries for selected portions of the animation sequence simultaneously. Thick boundaries imply greater uncertainty for hardiness zone assignment whereas thinner boundaries imply less uncertainty for that particular portion of the map. We found this method to be useful for assessing boundary variability, but less useful for detecting the differential occurrence of data islands.

"We found [flickering] to be useful for assessing boundary variability, but less useful for detecting the differential occurrence of data islands."

Information Stability

Information stability was assessed by recording the variability in hardiness zone assignment for 68 sample points by interpolation method and gridding interval. The sample includes capital cities within the 48 conterminous states, as well as 20 additional point locations. The location of capital cities was determined according to coordinates provided by the

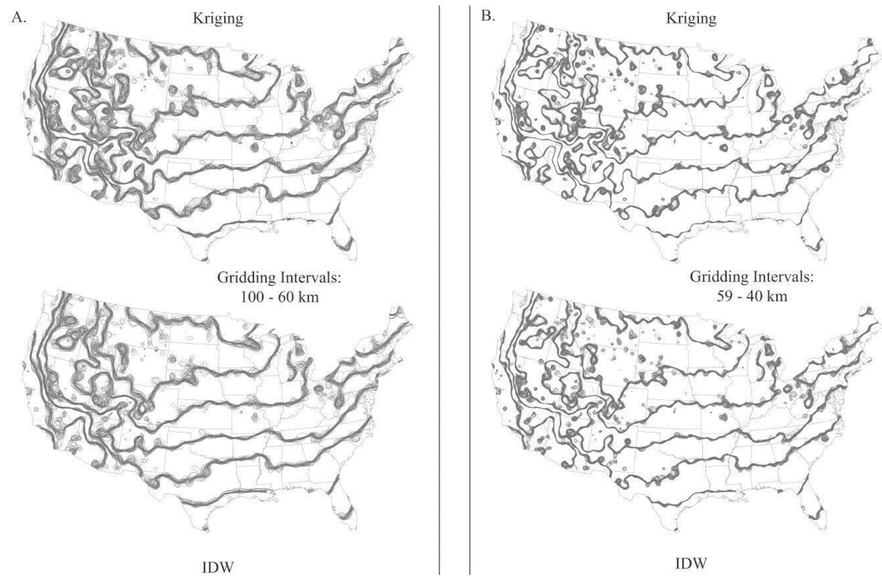

Figure 5. Composite maps showing the variability in hardiness zone boundaries for kriging and IDW for selected portions of the animation sequence (see Animations 1-3). Greater boundary width indicates greater method-produced uncertainty. a) Gridding intervals from 100-60 km. b) Gridding intervals from 59-40 km.

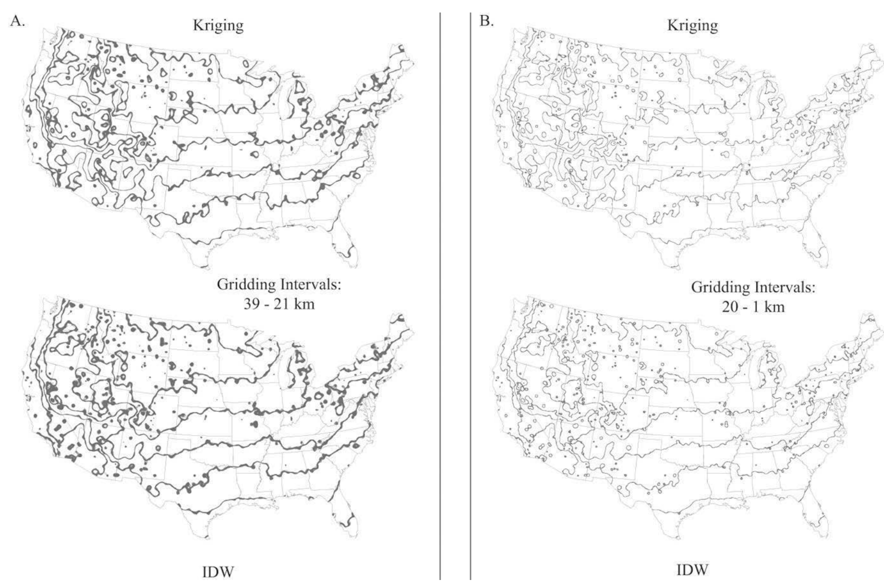

Figure 6. Composite maps showing the variability in hardiness zone boundaries for kriging and IDW for selected portions of the animation sequence (see Animations 1-3). Greater boundary width indicates greater method-produced uncertainty. a) Gridding intervals from 39-21 km. b) Gridding intervals from 20-1 km.

National Atlas of the United States (USGS, 2006). Because all capital cities fall within 10 km of the control points, 20 additional sample points were chosen based on their distance from control points, with 5 points selected at distance ranges of 10-30km, 30-50km, 50-70km, and >70km.

The plant hardiness zone that was assigned to each sample point was assessed for all 354 maps. These data were used to calculate hardiness zone changes as gridding interval becomes finer. By zone change, we mean a shift in zone assignment between sequential animation frames. For example, if a sample point was assigned zone 4, then zone 5, and then zone 4; two zone changes would be recorded. If a sample point was assigned zone 4, then zone 5, then zone 5; only one zone change would be recorded.

Of the 68 sample points, 24 (35%) changed zones at least once, 16 (24%) changed zones over three times, and 5 (7%) changed zones over fifty times. Interestingly, Pierre, South Dakota changed zones 120 times, and Salt Lake City, Utah changed zones 90 times (Figure 7a). Table 2 shows the number and percent of control points that changed zones at least once according to 10-frame groupings. Figure 7b shows that the information contained in the plant hardiness zone maps tends to become more stable as gridding interval decreases, but also suggests that this relationship is not necessarily straightforward or predictable until stability occurs.

". . . Pierre, South Dakota changed zones 120 times, and Salt Lake City, Utah changed zones 90 times. "

Discussion

Our results shed light on some interesting trends regarding the behavior of Kriging versus IDW for plant hardiness zone boundaries when different gridding intervals are employed in the interpolation process. With respect to interpolation accuracy, as indicated by RMSE values, kriging appears to perform more accurately at coarser intervals, while IDW tends to perform slightly better at intervals finer than 9 km (see Figure 2, Table 1). This difference, however, is minor, with the greatest difference in RMSE being 0.35˚ F at a gridding interval of 42.9 km (see Figure 4 for significance).

In terms of visual uncertainty, for both kriging and IDW, hardiness zone boundaries tend to stabilize at a gridding interval of ~10 km when the maps are viewed at a scale of ~1:22,000,000. Further research is necessary to know if "average" map viewers would consider the maps to be visually stable when gridding intervals of less than 10 km are employed. Furthermore, this information could be used to explore the relationship between

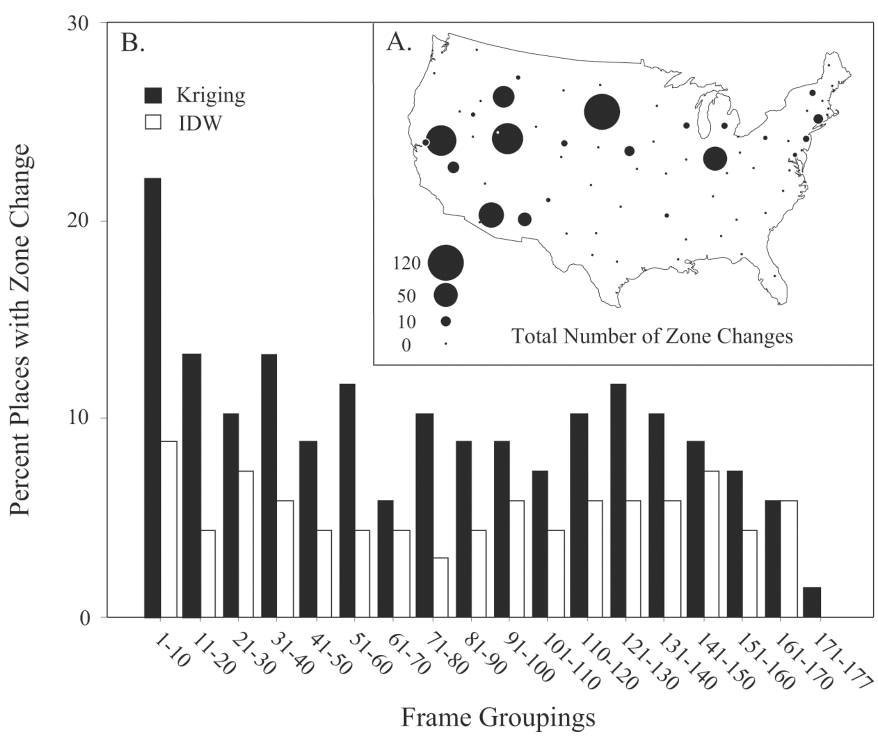

Figure 7. Information stability of hardiness zone assignment for 68 sample locations. (a) Total number of zone changes. (b) Percent places having zone changes by interpolation method according to 10-frame groupings in the animation sequence (see Animations 1-3).

Animation Frame #	Gridding Intervals (km)	**Kriging Zone Changes**		**IDW Zone Changes**	
		Number	Percent	Number	Percent
1-10	100.8 - 84.3	15	22	6	9
11-20	82.8 - 71.3	9	13	3	4
21-30	70.2 - 61.8	7	10	5	7
31-40	61.0 - 54.5	9	13	4	6
41-50	53.9 - 48.8	6	9	3	4
51-60	48.3 - 44.2	8	12	3	4
61-70	43.8 - 40.3	4	6	3	4
71-80	40.0 - 37.1	7	10	2	3
81-90	36.8 - 34.3	6	9	3	4
91-100	34.1 - 32.0	6	9	4	6
101-110	31.8 - 29.9	5	7	3	4
110-120	29.7 - 28.1	7	10	4	6
121-130	27.9 - 26.5	8	12	4	6
131-140	26.3 - 25.0	7	10	4	6
141-150	24.9 - 23.7	6	9	5	7
151-160	23.6 - 18.0	5	7	3	4
161-170	17.0 - 8.0	4	6	4	6
171-180	7.0 - 1.0	1	1	0	0

Table 2. Number and percent of 68 sample locations having zone changes by 10-frame groupings in the animation sequence (see Animations 1-3; Figure 6).

interpolation accuracy (as expressed as RMSE) and visual stability. While empirical studies with human subjects are outside the scope of this paper, such research should be explored in the future. In terms of information content, IDW appears to produce much more stable boundaries than Kriging. The overall difference in patterning provided by IDW and Kriging further emphasizes the tendency for IDW to create isolated data islands when all control point data are used for interpolation, a trend noted by Slocum (1999).

". . . there is no clear relationship between the average distance between control points (~46 km) and the gridding interval at which visual stability tends to occur (~10 km)."

Interestingly, there is no clear relationship between the average distance between control points (~46 km) and the gridding interval at which visual stability tends to occur (~10 km). Furthermore, the sample points that showed the greatest information instability do not necessarily take place near a relatively consistent isoline boundary. For example, Salt Lake City, Utah and Pierre, South Dakota have a high number of zone shifts because isolated data islands appear and disappear repeatedly, even when only slightly different gridding intervals are employed.

The results reported here support the assertion that summary statistics, as used for evaluating interpolation accuracy, are not sufficient for characterizing the uncertainty associated with isometric mapping (see Declerq, 1996). When we applied a gridding interval of 9 km, kriging and IDW produced interpolated grids having nearly identical RMSE values (~0.73). The patterning shown in the two isometric maps, however, is quite different (e.g., Mohave Desert; Figure 8). This holds true even when the same interpolation method is applied with different gridding intervals, especially when gridding intervals are relatively large. For example, when kriging is applied at intervals of 100 km and 99 km, dramatically different maps result, but RMSE values differ by only 0.006˚ F. The difference between these two maps may seem trivial if only RMSE is considered.

"The relationship between method-produced uncertainty and gridding interval is not as straightforward as that suggested by the systematic decrease in RMSE values shown in Figure 2."

The relationship between method-produced uncertainty and gridding interval is not as straightforward as that suggested by the systematic

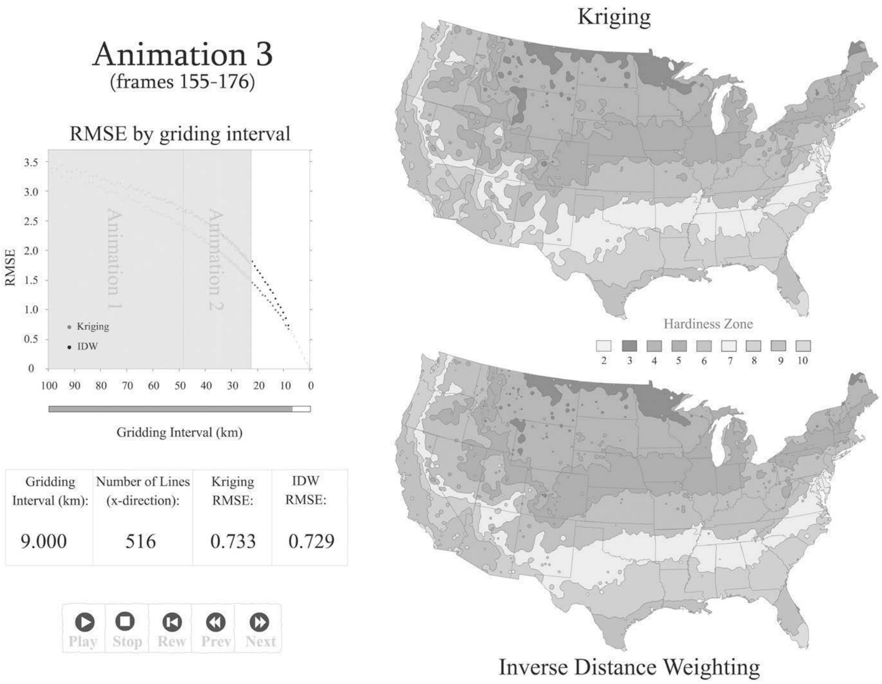

Figure 8. Screen capture of frame 169 of Animation 3 showing the difference in patterning for kriging and IDW when RMSE values are nearly identical. (see page 74 for color version)

decrease in RMSE values shown in Figure 2. While finer gridding intervals tend to be associated with greater information stability, the percent of zone changes fluctuates chaotically until stability occurs (see Figure 7, Table 2). For example, when IDW is employed, a greater number of sample points demonstrate zone change at intervals ranging from 24.9-23.7 km (Animation Frames 141-150), than at intervals ranging from 82.8-71.3 km (Animation Frames 11-20).

Given these results, we might naturally recommend that cartographers employ the smallest gridding interval possible for interpolation so that more stable isometric maps will result. The fact that kriging and IDW show stable but very different representations of hardiness zone boundaries at fine gridding intervals supports the assertion that stability does not necessarily equal truth (but see MacEachren, 1995; Muehrcke, 1990). Moreover, changes in a variety of other parameters, such as search sector size, weighting exponent, or semi-variogram model, may produce even different patterning when fine gridding intervals are used in the interpolation process. Our point is that no single isometric map will necessarily best represent hardiness zone boundaries because isometric maps contain inherent properties that make it difficult to verify a map's true accuracy. Rather than attempting to make a single best hardiness zone map, we focus on communicating the nature and magnitude of hardiness zone uncertainty by showing map readers a variety of reasonable hardiness zone maps, as discussed below.

". . . no single isometric map will necessarily best represent hardiness zone boundaries because isometric maps contain inherent properties that make it difficult to verify a map's true accuracy."

Communicating Method-Produced Uncertainty in Plant Hardiness Zones

We examine the use of flickering and transparency for communicating method-produced uncertainty in plant hardiness zones in order to examine their effectiveness when applied to our set of data. First, we created a non-temporal animation that flickers a variety of reasonable hardiness zone maps (Animation 4 [http://www.nacis.org/index.cfm?x=24]; Figure 9). Our intention is to allow map users to differentiate between hardiness zone patterning that is method-produced and hardiness zone patterning that is data-produced. To assemble the animation, we chose 11 hardiness zone maps for each interpolation method. The animation shows hardiness zone maps from every tenth frame of the original animation sequence (Animations 1-3), with gridding intervals ranging from 25-54 km. Controls are included to allow users to toggle between the kriging and IDW animated sequences. Map users are not allowed to stop the animation so that more confidence cannot be placed in any single hardiness zone map.

While we found flickering to be a very effective technique for visualizing isometric uncertainty, this technique requires viewing the map on a computer screen. As an alternative to flickering, we explore the use of transparency for displaying several hardiness zone maps simultaneously. Figure 10 is a composite of 354 plant hardiness zone maps, half produced using kriging, and the other half using IDW. The composite map was created by stacking all 354 plant hardiness zone maps, deleting polygon outlines, and displaying each map at a 99% transparency. The premise behind this application of transparency is that a mixture between colors along the spectral sequence relates to method-produced uncertainty. Colors that do not deviate from the conventional plant hardiness zone color scheme

"The composite map was created by stacking all 354 plant hardiness zone maps, deleting polygon outlines, and displaying each map at a 99% transparency."

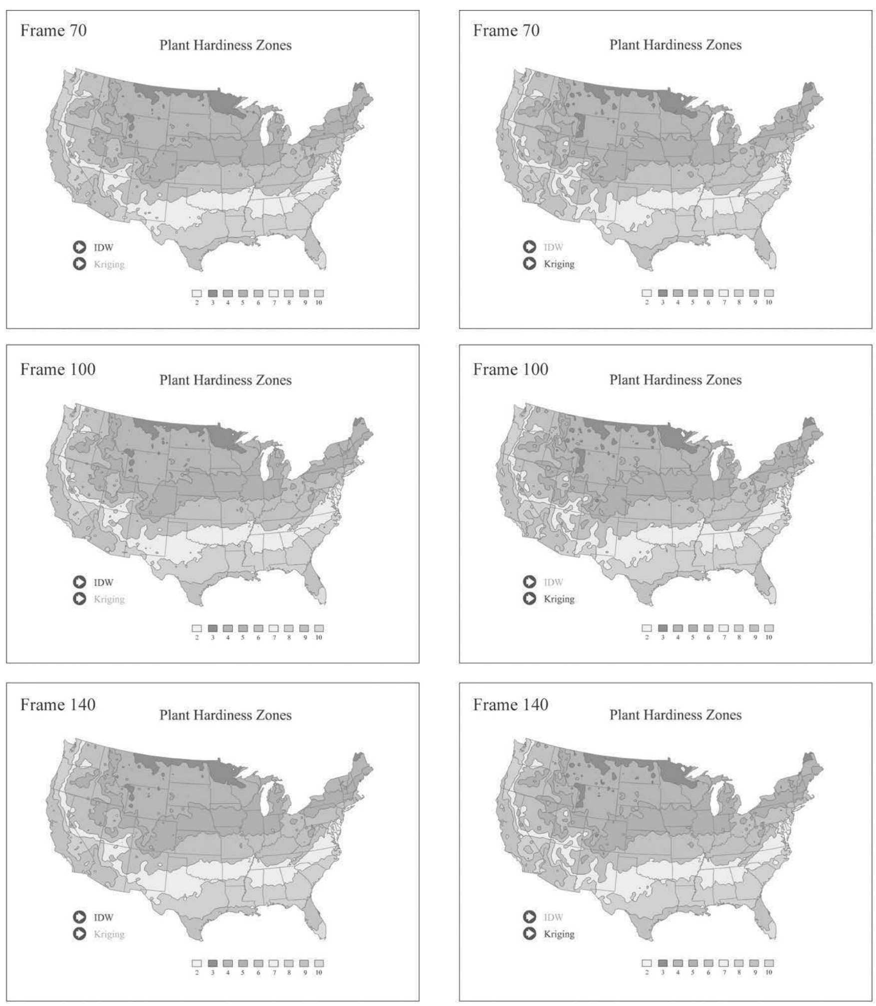

Figure 9. Selected frames from Animation 4. Frames on the left are IDW; frames on the right are kriging. (see page 75 for color version)

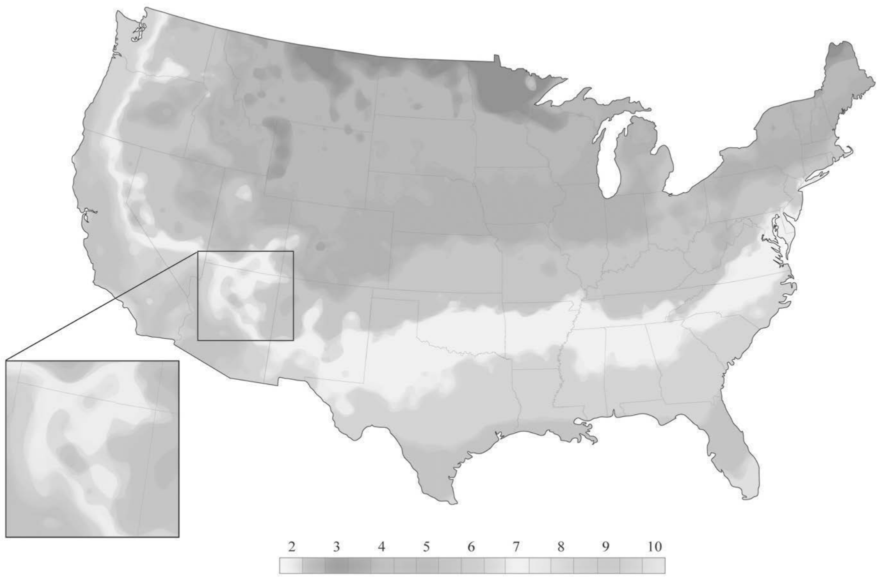

Figure 10. Composite map of 354 isometric representations of plant hardiness zones using kriging and IDW interpolation methods. (see page 75 for color version)

(e.g., green, yellow, orange) indicate places that are consistently assigned the same hardiness zone. In contrast, "mixed" hues (e.g., greenish-orange, yellow-orange) indicate places that are repeatedly assigned different hardiness zones. Rather than showing the results of just one interpolation method, our intention is to allow users to assess portions of the map where both kriging and IDW defined hardiness zones similarly, and places where they defined hardiness zones differently.

Conclusion

Cartographers have long recognized that all maps have a degree of uncertainty. We know that the conceptual and methodological decisions made during the mapping process can greatly affect the visual outcome of a particular map representation. While the entire process of map creation is an uncertain endeavor, the method-produced uncertainty associated with isometric mapping is exacerbated by the fact that the majority of data values shown on a map are predictions that cannot be verified. Even though cartographers know that isometric maps are highly uncertain and therefore, prone to misunderstandings, the exact nature and magnitude of this uncertainty for a particular map may be often unexplored, and is rarely conveyed to map users.

". . . our intention is to allow users to assess portions of the map where both kriging and IDW defined hardiness zones similarly, and places where they defined hardiness zones differently."

In using plant hardiness zones as a case study, we have attempted to show that when different methodological decisions are made in the interpolation process, very different maps can result. For plant hardiness zone maps, these visual and informational differences are not easily explained, and are not as predictable as that suggested by interpolation accuracy statistics. Despite these properties, isometric mapping continues to be a preferred method for many social and physical scientists, particularly in the fields of climatology and meteorology, where isometric maps are often used for analytic purposes, or to assist in policy decision-making. Use of the exploration and visualization techniques examined herein might direct map users to a better understanding of uncertainty about isometric map interpolation through visual, as opposed to numerical, means. We believe that not only could these techniques assist map users in understanding uncertainty, they may also allow map users to feel more certain, when certainty it is warranted.

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